# MATHEMATICAL PRODIGIES.[1]

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The object of the present paper is threefold:

(1) To give a summary of the mathematical prodigies[2] described in the literature of the subject, without, however, duplicating unnecessarily the work of previous writers.

(2) To give a brief account of the writer's own case, which is, it is believed, fairly typical, despite certain peculiar limitations to be described later, and which will shed light on certain factors in mental calculation that have not hitherto received full recognition.

(3) To set forth a new theory of mental calculation, based upon the foregoing data, and incidentally to criticise certain other theories hitherto advanced in this field.

## I.

In view of the incompleteness of existing data in most cases, and the inaccessibility of some even of the existing sources of information, a complete history of the mathematical prodigies would be out of the question. We shall, therefore, simply attempt to give a reasonably complete list of those of whom definite information is available, together with a statement of the significant facts known about them. A few names—that of Euler, for example—have been omitted on account of the absence of any satisfactory data that would shed light on the theory of mental calculation; and no attempt has been made to collect the accounts of new prodigies found every now and then in the newspapers. Such accounts are not readily accessible, and

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[1] From the Psychological Seminary of Cornell University.

[2] By a "mathematical prodigy" we shall mean a person who shows unusual ability in mental arithmetic or mental algebra, especially when this ability develops at an early age, and without external aids or special tuition. We shall use the word "calculator" in the sense of "mental calculator," as a synonym for "mathematical prodigy," and shall usually mean by "calculation" "mental calculation," unless the contrary is clearly indicated by the context. A "professional calculator" will be taken to mean a mental calculator who gives public exhibitions of his talent. "Computer," however, will be restricted to mean one who calculates on paper. All problems mentioned as solved by the mathematical prodigies will be understood to be done mentally, unless otherwise indicated.

are usually so popular and unreliable that they have little scientific value.

There are several possible bases for a classification of the mathematical prodigies. We might group them chronologically, as Scripture[1] does; or by the extent of their power, as measured either by the size of the numbers they could handle or by the rapidity of their calculations; or by the degree of their mathematical ability, as shown by the character of the problems they solved and the processes they used. Or we might classify them according to memory type, as either visual or auditory calculators. No one of these classifications, adhered to consistently throughout, would quite answer the purpose here, owing to the great unevenness of the material at hand in the case of the different calculators. An arrangement has therefore been adopted which is in part chronological, but which is modified by most of these other considerations. In this way, so far as the crossing of the different principles of division permits, those men are in the main brought together who are most naturally compared, and the important points of resemblance and difference come out more conveniently than if an abstractly logical arrangement were adopted.

We begin, then, with Fuller and Buxton, who have much in common, and who are the first modern calculators about whom reliable data are available. Colburn, Mondeux, and Inaudi form the next group, followed by Zaneboni, Diamandi, and Dase. Then come the two Bidders and Safford, followed by Gauss and Ampère, and finally those who may be called "minor prodigies," whether because of limited powers of calculation or because the available information is not sufficient for a more detailed account.

*Tom Fuller*[2] (1710-1790), "the Virginia calculator," came from Africa as a slave when about 14 years old. We first hear of him as a calculator at the age of 70 or thereabouts, when, among other problems, he reduced a year and a half to seconds in about two minutes, and 70 years, 17 days, 12 hours to seconds in about a minute and a half, correcting the result of his examiner, who had failed to take account of the leap-years.[3] He also found the sum of a simple geometrical pro-

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[1] In his article on "Arithmetical Prodigies," in the *American Journal of Psychology*, IV, 1891, pp. 1-59. We shall hereafter have frequent occasion to refer to this article, the only one in English in which a comprehensive study of the subject is attempted.

[2] Scripture, *op. cit.*, p. 2; Binet, *Psychologie des grands calculateurs et joueurs d'échecs*, 1894, p. 4; *American Museum*, V, 1789, p. 62. This last date is erroneously given by Scripture as 1799.

[3] Binet, *op. cit.*, p. 5, notes that the harder problem was done in less time than the simpler one, and is inclined to suspect that the records are unreliable. But in the case of so slow and plodding a calculator

gression, and multiplied mentally two numbers of 9 figures each. He was entirely illiterate.

*Jedediah Buxton*[1] (1702-1772) was very stupid even from boyhood. Though his father and grandfather were men of some education, he remained illiterate all his life, and was of less than average intelligence; even the statement of a mathematical problem he comprehended, we are told, "not without difficulty and time." In calculation he was, like Fuller, extremely slow; but he had a prodigious memory, and could retain long numbers for days or even months, so that he performed enormous calculations, which in some cases occupied him for weeks. On one occasion he mentally squared a number of 39 figures, in 2½ months. His methods were original, but very clumsy; to multiply by 378, in one instance, he multiplied successively by 5, 20, and 3 to get 300 times the number, then by 5 and 15 to get a second partial product, and finally by 3, to complete the operation. Thus instead of adding two zeros to multiply by 100, he multiplied first by 5 and then by 20. This fact, together with his slowness, shows pretty clearly that his methods were of counting rather than multiplication, though we are told that he had learned the multiplication table in his youth. He could give from memory an itemized account of all the free beer he had had from the age of 12 on. He was able to calculate while working or talking, and could handle two problems at once without confusion. At a sermon or play Buxton seems to have paid no attention to the speaker's meaning, but to have amused himself by counting the words spoken, or the steps taken in a dance, or by some long self-imposed calculation. He could call off a number from left to right or from right to left with equal facility, and by pacing a piece of ground could estimate its area with considerable accuracy.

*Zerah Colburn*[2] (1804-1840), the son of a Vermont farmer,

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as Fuller, little importance can be attached to such discrepancies, especially since the times given are only approximate. Moreover, Fuller was at this time about 70 years old himself, and may therefore have had in his memory, already calculated, the number of seconds in 70 years. The times given seem to indicate that he used a process of modified counting, rather than multiplication in the ordinary sense. The importance of this distinction will appear later.

[1] Scripture, *op. cit.*, p. 3; *Gentleman's Magazine*, XXI, 1751, pp. 61, 347; XXIII, 1753, p. 557; XXIV, 1754, p. 251.

[2] Also spelt *Colborne*. Scripture, *op. cit.*, p. 11; *A Memoir of Zerah Colburn, written by himself*, Springfield, 1833; *Philosophical Magazine*, XL, 1812, p. 119; XLII, 1813, p. 481; *Analectic Magazine*, I, 1813, p. 124; Carpenter, *Mental Physiology*, §205, p. 232; *Cornhill Magazine*, XXXII, 1875, p. 157; *Belgravia*, XXXVIII, 1879, p. 450; Gall, *Organology*, §XVIII, pp. 84-7 (in *On the Functions of the Brain*, V, Eng. tr., Boston, 1835). Scripture gives two other references which

was regarded as a backward child until the end of his 6th year, when one day his father heard him repeating parts of the multiplication table, though the boy had had only about six weeks' schooling. The father then "asked the product of 13 x 97 to which 1261 was instantly given in answer. He now concluded that something unusual had actually taken place; indeed he often said he should not have been more surprised, if some one had risen up out of the earth and stood erect before him." [1] The elder Colburn now took Zerah about the country, giving public exhibitions of the child's powers in various cities. Colburn was thus the first professional calculator, in the sense already defined. From the list of questions answered by him at Boston, in the fall of 1810, and from the account in the body of the *Memoir*, it appears that even at this early date, only four months after the discovery of his talent, he was a good calculator, though of course he improved with further practice. It is clear, therefore, that his powers had been developing for some time—to judge from other cases at least six months, if not a year—before they attracted his father's attention. This may mean that he learned to count from his elder brothers and sisters,—the eldest was about seven years older than Zerah,—rather than from his own brief six weeks at school. Colburn's preference for multiplication, the extraction of roots, factoring, and the detection of primes seems to have developed early; he never became as proficient in division as Bidder, for example, and, like most of the prodigies, he used addition and

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the writer has been unable to consult: *The Amerian Almanac*, 1840, p. 307, and the *Medical and Philosophical Journal and Review*, III, 1811, p. 21. Gall's account, however, seems to be based upon this last article.

[1] *Memoir*, pp. 11-12. Scripture (*op. cit.*, p. 12) is "tempted to ask for the authority on which the statements were made", and inclined not to "put too much faith in the figures", on the ground that Colburn never speaks of himself as having any extraordinary power of memory for long periods of time. But the full passage as quoted above makes it clear that the father had told the incident repeatedly to awe-stricken listeners in Zerah's hearing; moreover, the remembering of such a simple problem could hardly require "extraordinary power of memory" in a person used to mental calculation. Colburn's feats in factoring large numbers are hard to explain except by supposing that he remembered at least those numbers which he had previously examined and found prime. This would imply a rather considerable development of his memory for figures. At any rate, there is nothing improbable in his remembering the figures quoted in the text, even for some years after his calculating powers had declined.

It may be noted that later in his article Scripture's faith in Colburn's memory increases; for on page 46 he thinks we can presuppose in the case of Colburn and certain others an extended multiplication table, perhaps even to 100 x 100. Reasons for rejecting this supposition, in Colburn's case at any rate, will appear later.

subtraction only incidentally, in the service of other operations, not for their own sake. In answering catch questions and in repartee he was moderately clever.

In the spring of 1812 Zerah was taken by his father to London. Here, among other feats, he found mentally, by successive multiplication, the 16th power of 8 (= 281,474, 976,710,656) and the 10th powers of other 1-figure numbers, also, though with more difficulty, the 6th, 7th, and 8th powers of several 2-figure numbers. The square root of 106,929 (= 327) and the cube root of 268,336,125 (= 645) were found "before the original numbers could be written down." He immediately identified 36,083 as a prime number, and found "by the mere operation of his mind" the factors, 641 and 6,700,417, of 4,294,967,297 ($=2^{32}+1$).[1]

While in London, Colburn learned to read and write, and later began the study of Algebra; but his education was subject to long interruptions, owing to the constant financial difficulties caused by his father's lack of business ability. After visits to Ireland and Scotland, the Colburns went, in 1814, to Paris, where Zerah spent eight months at school, studying mainly

[1] *Memoir*, pp. 37-8, quoting from a prospectus printed in London, 1813. From Colburn's own account of his methods of factoring (pp. 183-4), it appears that the only way in which he could *immediately* identify as prime such a number as 36,083 would be by remembering the result of a previous examination of it. Scripture (*op. cit.*, p. 14, note) says that it "requires considerable faith" to accept the statement that Colburn factored $2^{32}+1$. But we are *not* told that he did it "instantly"; a friend of Morse's says simply, "almost as soon as it was put to him" (Scripture, *loc. cit.*, quoting from a letter in S. I. Prime's *Life of Samuel F. B. Morse*, p. 68; the reference is undoubtedly to this problem), while Carpenter (*Mental Physiology*, p. 233; the writer has not been able to find Carpenter's authority for this statement) says, "*after the lapse of some weeks.*" Even if the time was only a matter of some minutes, the feat is not incomprehensible. The smaller factor, 641, might easily have been hit upon by a lucky trial at a very early stage of the work. We read in Baily's account (*Analectic Magazine*, I, 1813, p. 124) that "any number, consisting of 6 or 7 places of figures, being proposed, he [Colburn] will determine, with . . . expedition and ease, all the factors of which it is composed." Now $2^{32}+1$ is only a 10-figure number, or three figures longer than those Colburn was used to handling; and the smallness of the factor 641 renders the problem much simpler than it at first appears. Since, then, the feat is entirely possible, and since it is cited by Colburn from the publicly circulated Prospectus of 1813, and is mentioned by at least one contemporary writer who was not acquainted with the *Memoir*, there is no reason for believing that Colburn fabricated the incident; especially since his limited mathematical knowledge would never have shown him the importance of this particular number. Had he been inventing out of whole cloth, he would have multiplied together two prime numbers chosen at random, and would probably have made the smaller one at least a 4-figure, if not a 5-figure number. On the historical reliability of the *Memoir* see Appendix I.

French and Latin. Returning to England early in 1816, he entered Westminster School in September, under the patronage of the Earl of Bristol, making fair progress in the languages, and standing well in his class, in which, however, he was one of the oldest boys. He also studied six books of Euclid under a private tutor, but showed no marked geometrical aptitude. In 1819 his father removed him from school, and soon after we find him, at his father's suggestion, unsuccessfully attempting the career of an actor and playwright. In 1822 he opened a small school, which ran for a year or more. His next occupation was as a computer in the service of the secretary of the Board of Longitude. Shortly after his father's death, in 1824, Zerah returned to America, and in December of 1825 joined the Methodist church, becoming a circuit preacher. After seven years of this occupation,[1] being in need of funds to eke out his modest ministerial salary, he wrote the *Memoir*, carrying out a plan which his father and friends had had in view long before. In 1835 he resumed teaching, as "Professor of the Latin, Greek, French and Spanish Languages, and English Classical Literature in the seminary styled the Norwich University."[2] He died in 1840.

From this brief account of Colburn's romantic career, it will be seen that his education, while much interrupted, was fairly good. He spent four or five years in the study of languages, for which he seems to have had a natural liking, and later was able to teach them. He began the study of algebra, but did not get beyond the elements of it; and he studied geometry, which he found easy but uninteresting, owing to the lack of any visible practical application. The literary style of his *Memoir*, though far from Addisonian, is always readable, the book is interesting throughout, and even the specimens of his poetry given in the appendix are not specially bad, all things

[1] *I. e.*, in 1832 or 1833. *Cf. Memoir*, p. 31, "after possessing the talent twenty-two years", from August, 1810; p. 142, "nine years' residence here" in America, from June, 1824; p. 166, "twenty-two years ago", to 1810 or 1811; p. 167, "the last seven years that he has spent in the traveling connection", from December, 1825. These passages show that the *Memoir* was not begun, or at any rate had not reached the third chapter, before 1832, and was not completed until shortly before its publication in 1833. Scripture's statement, therefore (*op. cit.*, p. 11, note [2]), that "there is no statement regarding the time at which they [the *Memoir*(s)] were written, or even a date to the preface; the last year mentioned in the book is 1827", is decidedly misleading. The last date printed in figures, to be sure, so that it could be identified by a cursory glance, is 1827; but the last date "mentioned" is certainly 1832, if not 1833, even granting that all the periods of time above quoted are only approximate, and cannot be taken without an allowance of half a year one way or the other for possible error.

[2] Scripture, *op. cit.*, p. 16, quoting from *American Almanac*, 1840, p. 307.

considered. The question of the historical reliability of the *Memoir* will be discussed later; for the present it will suffice to say that, on a careful reading, the book shows scarcely a trace of that self-glorification with which it has been charged by Scripture and Binet.

Concerning the rapidity of Colburn's calculations not much is known. The only series of problems whose times he gives us dates from 1811, before he was 7 years old, and so is hardly typical of his performances two or three years later when he was in his prime. The times indicated are fairly short, in most cases shorter than if the work had been done on paper by a good computer. The testimony of observers as to his "extraordinary rapidity" is of little value in the absence of definite figures; especially since some of his feats, notably the extraction of square and cube roots and the finding of factors, were accomplished by the aid of extremely simple methods. Colburn's powers probably increased up to the time of his visit to Paris in 1814; but when he gave up his regular exhibitions, and became interested in other matters, he gradually lost much of his skill. There seems to be no authority, however, for the statement[1] that after a time his powers left him entirely; in 1823, at any rate, after a considerable period of disuse, they were readily revived for purposes of written longitude computations.

Of his methods of calculation Colburn has left us a very good account; the only calculator of whom we have a fuller account is Bidder,[2] whose methods closely resembled Colburn's. Both men, in multiplication, began at the left, instead of at the right as we usually do in written computations; and both, by the aid of certain properties of the 2-figure endings[3] of the

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[1] Scripture, *op. cit.*, p. 15.

[2] Bidder's account is more detailed, better written, and in more concise mathematical language than Colburn's, as a result of Bidder's superior educational advantages; it contains, furthermore, explanations of several of Bidders's feats, such as the solving of compound interest problems, which would have been hopelessly beyond Colburn's powers. At the same time Colburn's account is perfectly clear, to the non-mathematical reader perhaps even clearer than Bidder's. In this matter, as in several others, Scripture is hardly fair to Colburn; thus he speaks of Colburn's explanations as "the least intelligible of all the explanations" (p. 50). It is no reproach to Colburn that he was excelled by Bidder; but he certainly deserves credit for what he did do, and one of the things he did was to write a very good account of his methods, over twenty years before Bidder followed his example.

[3] By a 2-figure ending we shall mean the last two figures of any given number; thus 56 is the 2-figure ending of 3456, 01 of 2401, 07 of 7, etc. What properties of these endings were used by the mental calculators will be explained hereafter.

numbers used, were able to find with remarkable ease and rapidity the square and cube roots of exact squares and cubes, and also, though less rapidly, the factors of fairly large numbers.

Colburn had two physical peculiarities that need to be mentioned. (1) He possessed an extra finger on each hand and an extra toe on each foot. This peculiarity he shared with his father and two [1] of his brothers. (2) In his early years his calculations were accompanied by certain bodily contortions, similar to those of St. Vitus' dance. They seem to have passed away rather early; Colburn himself has no recollection of them, and mentions them simply on the authority of persons who saw him when "quite a child."[2]

*Henri Mondeux*[3] (1826-1862) was the son of a woodcutter near Tours. Sent to tend sheep at the age of 7, he amused himself by playing with pebbles, and thus learned mental arithmetic. Jacoby, a schoolmaster at Tours, hearing of him, sought him out, offered to instruct him, and gave him his address in the city; but the boy's memory outside mathematics was so poor that he forgot both name and address, and found the schoolmaster only after a month's search. He received instruction in arithmetic and other subjects, and in 1840 was exhibited before the Paris Academie des Sciences. In the committee's report on him we are told that he "carries on readily in his head not only the various arithmetical operations, but also, in many cases, the numerical solution of equations; he devises processes, sometimes remarkable, for solving

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[1] Colburn says (*Memoir*, p. 72), "his father and two of his [father's] sons," while the account in the *Philosophical Magazine* (XLII, 1813, pp. 481-2) says Zerah and three of his brothers. It has been assumed in the text that Zerah did not count himself, and that the other writer counted him twice; this is the simplest way of reconciling the two statements. The peculiarity had been in the Colburn family, we are told, for several generations.

[2] *Memoir*, p. 173. Scripture does not refer to this second peculiarity; but since Colburn mentions another mathematical prodigy with a similar affliction, and since Safford showed a striking nervousness in his early calculations, it has seemed worth while to mention the matter. Gall, probably quoting from the *Medical and Philosophical Journal and Review* article already cited, seems to refer to this nervousness when he says (*op. cit.*, V, p. 86): "While he [Colburn] answers, it is seen, by his appearance, the state of his eyes, and the contraction of his features, how much his mind labors." Colburn was not quite 7 years old when seen by the writer of the article on which Gall's account is based. Gall himself, however, examined Colburn in Paris, probably in 1814. *Cf. Memoir*, pp. 76-7.

[3] Scripture, *op. cit.*, p. 21; *La grande Encyclopédie*, art. *Mondeux;* Cauchy's report on Mondeux, in *Comptes rendus hebdomadaires des séances de l'Academie des Sciences*, XI, 1840, pp. 840, 952; reprinted in *Oeuvres Complètes de Cauchy*, 1e Série, 1885, V, p. 493, and in Binet, *op. cit.*, pp. 14-22. The writer has been unable to consult the other references cited by Scripture.

a great number of different questions which are ordinarily treated by algebra, and determines in his own way the exact or approximate value of integral or fractional numbers which satisfy given conditions." More specifically, he finds powers of numbers by rules of his own discovery which are equivalent to special cases of the binomial theorem; he has worked out formulas for the summation of the squares, cubes, etc., of the natural numbers, and for arithmetical progression and other series; he solves simultaneous linear equations by a method of his own, and sometimes equations of higher degree, especially where the root is a positive integer; and he solves such problems in indeterminate analysis as finding two squares whose difference is a given number. He "knows almost by heart the squares of all whole numbers under 100." Learning a number of 24 figures, divided into four 6-figure periods, requires 5 minutes. He can solve a problem while attending to other things.

Mondeux's admirers hoped that he would one day distinguish himself in a scientific career; but this was not the case. Like his successor Inaudi, whom he closely resembles in several respects, he became a professional calculator; but he had no ability outside of mathematics, and even there his powers soon reached a limit beyond which they did not increase. He died in obscurity. If we may judge by the Academy report, he was almost the equal of Bidder in his insight into mathematical relations;[1] but on the numerical side he was far excelled by Inaudi, who could, for example, memorize 24 figures in half a minute, a feat for which Mondeux required 5 minutes.

*Jacques Inaudi*[2] (b. 1867), an Italian by birth, passed his early years, like Mondeux, in tending sheep. An anecdote which Binet regards as rather doubtful indicates a possible prenatal influence in the direction of calculation; otherwise there is nothing noteworthy in his heredity. His passion for figures began about the age of 6, and at 7 he could carry on mentally multiplications of 5 figures by 5 figures. His education is very slight; he did not learn to read and write until he was 20 years old. Outside of mental calculation he has no special ability; his memory for most things except figures is rather poor, and he is often absent-minded. At last accounts he was still a professional calculator, living by public exhibitions of his talent. He visited the United States in 1901-2,

[1] Just how much Mondeux owed to Jacoby's teaching is hard to say. The writer has been unable to consult Jacoby's *Biographie d'Henri Mondeux* or Barbier's *Vie d'Henri Mondeux;* Binet, however, who cites both these works, says that Jacoby's lessons were "sans grand succès." (*Op. cit.*, p. 14.)

[2] Binet, *op. cit.*, pp. 24-109, 199-204, *et passim.*

appearing in many of the larger cities, and is said to have been fairly well received by American audiences.

Telling on what day of the week a given date falls is one of his favorite problems. The reduction of years, months, etc., to seconds he accomplishes almost instantly, knowing by heart the number of seconds in a year, month, week, or day. He solves by arithmetic problems corresponding to algebraic equations of the first and sometimes of higher degree, also such problems as the resolution of a given 4- or 5-figure number into the sum of four squares. In these latter cases, however, he proceeds for the most part simply by trial, aided, of course, by his skill in calculation and his familiarity with many squares, cubes, and the like. At his regular performances the programme includes the subtraction of one 21-figure number from another, the addition of five 6-figure numbers, the squaring of a 4-figure number, the division of one 4-figure number by another, the extraction of the cube root of a 9-figure number and the 5th root of a 12-figure number, or such similar problems as may be proposed by the audience. As each number is announced he repeats it slowly to his assistant, who writes it on the blackboard and then reads it aloud, to make sure there is no mistake. Inaudi then repeats the number once more, after which he devotes himself to the solution of the problem, meanwhile making an occasional remark to keep the audience in good humor. Throughout the exhibition he faces the audience, never once looking at the blackboard. Actually he begins his calculation as soon as the numbers are given, and carries it on during the various repetitions of the numbers by himself and his assistant, so that by the time he seems to begin the solution he may be well advanced toward the answer. In this way he appears to work much more rapidly than he really does.

Inaudi is a well-marked instance of the auditory[1] memory type. When he thinks of numbers, in calculation or otherwise, he does not see them "in his mind's eye," as arrays of dots or other small objects, or as written or printed figures; numbers are for him primarily *words*, which he hears as if spoken by his own voice, and during his calculations he almost always pronounces at least some of these words, either with partial distinctness or in a confused murmur. Any interference with

[1] Actually it would be more correct to call his type auditory-motor, and the same is probably true of most of the other auditory calculators we shall study, since a pure or non-motor auditory individual is rare. For convenience, however, the writer has followed Binet's terminology. The meagreness of our information in most cases makes it difficult to tell just what part the motor element plays; and this is especially true when we are dealing with a limited field like calculation, where the motor element may often play a less important part than in certain other fields.

this habitual articulation embarrasses him, and prolongs his calculation. He remembers a number very much more readily after hearing it than after seeing it; in fact, if a written number is handed to him, he usually reads it aloud, in order to learn it by sound rather than by sight. Whether visual images are *entirely* absent is a purely theoretical question; it is at least clear that, if present at all, they play a negligible part in his mental computations. We shall later find reason to believe that this condition is by no means so rare as has been supposed. Owing to the traditions of English and French psychology, the visual theory of mental calculation has lain ready to hand, and has in the past found much apparent confirmation. But now that an unmistakably non-visual calculator is on record, it will no longer do to beg the whole question; we must insist on considering each case upon its own merits, either settling it by definite evidence or leaving it frankly in doubt. We shall see later how much of the supposed evidence for the visual theory falls before a careful examination.

One of Inaudi's most marked characteristics is his powerful memory for figures. In one experiment he was able to repeat, after a single hearing, though with an effort, 36 figures, read off to him slowly in groups of three; but in the attempt to repeat 50 figures under the same conditions he became confused, and got only 42 of them correct. This latter number, 42, Binet therefore takes as the limit of Inaudi's power of acquisition, or "mental span," under these conditions. In an experiment made to determine in what time he could learn 100 figures read off to him in groups as often as requested, he learned the first 36 in a minute and a half, the first 57 in 4 minutes, 75 in 5½ minutes, and the whole 100 (actually there were 105) in 12 minutes. On the other hand, he can repeat in order, at any time within a day or two, all the figures used in his last performance, whether in the statement of the problems, in the answers, or in the intermediate calculations. The number of these figures at times runs as high as 300, and the total duration of the performance is usually not more than 10 or 12 minutes. Each new performance, however, blots out of his memory almost entirely the figures used in the previous one; but such constants as the number of seconds in a year, etc., as well as many powers and products, and any particular numbers or results in which he for any reason takes a special interest, remain permanently with him. These facts show how important it is to take account of the conditions of such experiments if the figures established by them are to have scientific value. In an experiment lasting the same length of time as one of his regular exhibitions, but under very different conditions, Inaudi can learn only a third the number of figures he

remembers with ease under his usual conditions. In these public performances, however, each number in the problem as given is repeated several times (twice by Inaudi himself, and once each by his assistant and the proposer of the question), and the figures of the various calculations and the result have a logical connection in the problem. Moreover, the numbers are learned in relatively short stages, separated by intervals in which they can be assimilated.[1]

Concerning the rapidity of Inaudi's calculations we have fairly full information,—so much fuller, in fact, than we have for any previous calculator, that no satisfactory comparisons can be made. Since the results of Binet's experiments are readily accessible, a brief summary of them will here suffice. In each experiment the subject was given a written column of numbers, each of which was to be mentally increased or diminished, multiplied or divided, by the same number; in other words, the addend, subtrahend, multiplier, or divisor was uniform for the whole given column of numbers. The results were called off down the column as fast as obtained, and the average time for each single operation thus determined. These tests were made on some of Binet's pupils, on Inaudi, and on four department store cashiers who were thoroughly practiced in addition, subtraction, and multiplication of small numbers, and could perform mentally 2-figure multiplications,[2] and in some cases, though with difficulty, 3-figure multiplications. The students were of course considerably slower than Inaudi and the cashiers; but the cashiers, in dealing with the smaller numbers to which they were accustomed, were fully as rapid as Inaudi, in some cases slightly more rapid. In dealing with larger numbers, however, which exceeded the limits of their customary calculations, their inferiority to Inaudi was very marked.

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[1] Mondeux, it will be remembered, required 5 minutes to learn 24 figures, whereas learning this number of figures is a common incident of Inaudi's exhibitions, and takes only half a minute. Here again, however, the results are not directly comparable. Mondeux learned the number in groups of 6 figures, and presumably from a paper or blackboard, while Inaudi always groups numbers in periods of three, and learns them by audition instead of vision. We shall refer later to a distinction which must be made between the direct and immediate remembering of figures which results from deliberately committing them to memory, and the very rapid and abbreviated automatic calculations which in some of the prodigies simulate direct memory. Recollection as the result of repeated calculation may form an intermediate stage in the passage of the latter into the former. These distinctions will become important in connection with the much discussed question whether, and to what extent, the mental calculators possessed extended multiplication tables.

[2] By a 2-figure, or $n$-figure, multiplication will be understood hereafter a multiplication in which each of the two numbers contains 2 (or $n$) figures, and the product 3 or 4 ($2n-1$ or $2n$) figures.

*Ugo Zaneboni*[1] (b. 1867), an Italian, born in the same year as his countryman Inaudi, received a fair education. His interest in numbers began at the age of 12, and when 14 he could solve[2] any problem his teacher proposed to him. While serving his term in the army he was for a time stationed at a railroad depot, where he amused himself by gradually committing to memory a vast body of statistics relating to time-tables, distances between different cities, population, tariffs, etc. When he later took to the stage as a professional calculator, questions based on these statistics formed part of his regular programme. Among his other usual feats are the repetition, either forwards or backwards, of a memorized number of 256 figures, the squaring of numbers up to 4 figures and the cubing of numbers up to 3 figures, finding the 5th powers of 2-figure numbers, and, conversely, extracting the 5th root of any number of 10 figures or less, the cube root of any 9-figure number, and the square root of any number of 7 figures or less, whether the given number is a perfect power or not. In these problems he is aided by his knowledge of many perfect squares, cubes, etc., as well as by various properties of 2-figure endings, with which he is thoroughly familiar. He possibly has a number-form, in which the numbers from 1 to 10, from 10 to 100, and from 100 to 1000 are arranged along three horizontal lines. This number-form, however, if it really exists, plays little or no part in his actual calculations.

*Pericles Diamandi*[3] (b. 1868), the son of a Greek grain merchant, attributes his calculating gift to his mother, who "has an excellent memory for all sorts of things." One brother and one sister, out of a family of fourteen, share his aptitude for mental arithmetic. He entered school at the age of 7, and remained there until he was 16, always standing at the head of the class in mathematics. But it was only after entering the grain business himself, in 1884, that he discovered his powers of mental calculation, which he now found very useful. He knows five languages,—English, French, German, Roumanian, and his native Greek,—and is a great reader; he has read all he can find on the subject of mental calculation; and he has written novels and poetry, concerning whose quality, however, Binet does not enlighten us. It will thus be seen that Diamandi's education is much better than Inaudi's, and his range

[1] *Rivista sperimentale di Freniatria*, etc., XXIII, 1897, pp. 132-159, 407-429. A summary of these articles, in German, is found in the *Zeitschrift für Psychologie und Physiologie der Sinnesorgane*, XVI, 1898, p. 314. The writer is indebted to Mrs. Rose Harrington for a translation of considerable portions of the original Italian articles.

[2] Mentally, it is to be presumed, though the article is not explicit on this point.

[3] Binet, *op. cit.*, pp. 110-154, 98, 187 ff. *passim*.

of interests correspondingly wider, but that he was far less precocious in calculation than his rival.

Diamandi is of the visual memory type. He has a number-form of a common variety, running zigzag from left to right, and giving most space to the smaller numbers. This number-form he sees as localized within a peculiar grayish figure, which also serves as a framework for any particular number or other object which he visualizes. He has colored audition for the names of various persons, the days of the week, etc., and if a few figures in a given number differ in color from the rest, he remembers the colors without effort. If the color scheme is more complicated, however, he first memorizes the number and then learns the colors of the individual figures. He always sees numbers as written in his own handwriting, and preferably, if the numbers are large, in a rectangle as nearly square as possible, rather than in one or two long lines. He learns spoken figures (in French) much less readily than written, since in the case of spoken figures he must not only call forth the corresponding visual images, but translate the numbers into his native Greek, in which all his calculations are carried on. Where he seeks to learn the figures very accurately, for purposes of calculation, he is only about half as fast as Inaudi;[1] but where he is concerned with speed rather than accuracy his times are much shorter. In the one case he learned 10 figures in 17 seconds; in the other, 11 figures in 3 seconds.

In calculation Diamandi is considerably slower than Inaudi, whether the numbers concerned are large or small. His time was 127 seconds for a 4-figure multiplication, whereas Inaudi could accomplish the same feat in 21 seconds. Diamandi finds the various figures of the product in order, from right to left, by cross-multiplication; thus in such an example as

$$\begin{array}{r}
46273 \\
729 \\
\hline
416457 \\
92546\phantom{0} \\
323911\phantom{00} \\
\hline
33733017
\end{array}$$

he finds the figures of the partial products not in the horizontal lines of the ordinary method, but in vertical lines,—first

[1] Here again, however, we must be careful about direct comparisons of dissimilar data, since Diamandi learned from a paper and wrote out his results, while Inaudi depended on audition and speech. Moreover, Diamandi's times were found to be subject to considerable variation from day to day.

7, then 5, 6, then 4, 4, 1, then 6, 5, 1, etc.,—and adds each column before he proceeds to find the numbers that compose the next column. This method has the advantage that the various figures of the partial products can be forgotten almost as fast as obtained, since that figure of the total product which depends on a given column of the partial product is found and recorded as soon as the column is known, and the numbers in that column therefore play no further part in the calculation. On Diamandi's performances in other operations than multiplication Binet gives us no data.

*Johann Martin Zacharias Dase*[1] (1824-1861) was born in Hamburg. Concerning his heredity we have no information. He attended school at the age of 2½ years, but attributed his powers to later practice and industry rather than to his early instruction. He seems to have been little more than a human calculating machine, able to carry on enormous calculations in his head, but nearly incapable of understanding the principles of mathematics, and of very limited ability outside his chosen field. In this respect he resembled Buxton; but in the rapidity and extent of his calculations he was incomparably superior to Buxton, or indeed to any other calculator on record. He multiplied together mentally two 8-figure numbers in 54 seconds, two 20-figure numbers in 6 minutes, two 40-figure numbers in 40 minutes, and two 100-figure numbers in 8¾ hours; he could extract the square root of a 60-figure number in an "incredibly short time," and the square root of a 100-figure number in 52 minutes. All these times, with the exception of that for the 100-figure multiplication, are probably more rapid, in some cases much more rapid, than those of a good computer using paper. Buxton, it will be remembered, once succeeded in multiplying two 39-figure numbers; other calculators, however, seem to have been unable to handle multiplications much above 15 figures. But if there was any definite limit to Dase's powers, the experiments of which we have record do not show it. We shall later find reason for believ-

[1]Also spelt *Dahse*. The full name is given on the authority of Brockhaus's *Konversations-Lexikon*, ed. 1898, art. *Dase*. Scripture, following the title-page of Dase's posthumously published *Factoren-Tafeln* (3 vols., 1862-5), gives the name as simply Zacharias Dase, which seems to be the way in which Dase usually wrote it. On Dase's life and calculations see Scripture, *op. cit.*, p. 18; *Briefwechsel zwischen Gauss und Schumacher*, Altona, 1861, III, p. 382; V, pp. 30, 32, 277-8, 295-8, 300-304; VI, pp. 27-8, 78, 112; *Crelle's Journal* (*Journal f. d. reine u. angewandte Mathematik*), XXVII, 1844, p. 198; Zacharias Dase, *Factoren-Tafeln*, Hamburg, Vol. I, 1862, Preface; Schröder, *Lexikon d. hamburgischen Schriftsteller*, 1851, art. *Dase;* Preyer, "Counting Unconsciously," *Pop. Sci. Monthly*, XXIX, 1886, p. 221; Brockhaus's *Konversations-Lexikon*, 1898, art. *Dase*. For other references see Scripture, *loc. cit.*

ing that the 100-figure multiplication was not really a severe tax upon his powers of mental arithmetic. In short, Dase's achievements so far transcend those of any other recorded calculator that he stands in a class by himself, unapproached by any of his rivals.

At the age of 15 Dase began his public exhibitions, and continued them for a number of years. He soon numbered among his friends several eminent mathematicians, however, and their influence gradually led him more and more to devote his vast powers to the service of science.[1] Among his (non-mental) computations are included the determination of the value of $\pi$ to 200 decimal places[2] by the formula

$$\frac{\pi}{4} = \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{5},$$

a labor of two months; the computation of the 7-place natural logarithms of the numbers from 1 to 1,005,000; and factor-tables for the 7th and 8th millions (except a small portion) and parts of the 9th and 10th millions. This last task, however, was one in which his patience and perseverance were of more value than his skill in calculation, since, by methods to which Gauss was careful to call his attention, the work was made mainly mechanical. Dase had planned to carry the table through the 10th million, but death cut short his labors. The

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[1] Scripture's statement (*op. cit.*, p. 19) that Colburn and Mondeux "enjoyed even greater advantages [than Dase,] yet failed to yield any results" in the service of science, is misleading. With both Mondeux and Dase the trouble seems to have been not lack of opportunity to acquire mathematical knowledge, but lack of native ability to use the opportunities they had. With Colburn, on the other hand, the trouble really was at least in part lack of opportunity; he certainly did not enjoy the opportunity to attend university lectures, nor did any eminent mathematician try "in vain for six weeks to get the first elements of mathematics into his head" (*ibid.*, p. 18; Gauss-Schumacher *Briefwechsel*, III, p. 382; V, pp. 32, 295), as in the case of Dase. Moreover, Colburn's description of his methods must be reckoned as an important contribution to the science of psychology, none the less important because it is somewhat inferior to Bidder's later description. For other instances of Scripture's unfairness to Colburn, see Appendix I.

[2] Scripture omits to mention any specific number of decimal places, though in both the references he gives (p. 18), to *Crelle's Journal* and to the Gauss-Schumacher *Briefwechsel*, the number of decimal places is made prominent. The natural inference would be that Scripture regarded $\pi$ as a commensurable number of exactly 200 decimal places; but in view of his frequent use of higher mathematics in his other published works, one hesitates to attribute to him so gross an error. Of course *anybody*, with a logarithm table and a little knowledge of geometry, can compute the value of $\pi$ to three or four places; the record of such a computation is absolutely meaningless without specific mention of the number of figures to which the computation is carried out.

tables were completed by another hand, and published as far as the 9th million in 1862-5.

Dase had one other notable gift, doubtless related to his calculating power: he could count objects with the greatest rapidity. With a single glance he could give the number (up to thirty or thereabouts) of peas in a handful scattered on a table; and the ease and speed with which he could count the number of sheep in a herd, of books in a case, or the like, never failed to amaze the beholder. Here, again, his powers are so far in advance of those of any other recorded person that he stands in a class by himself.

*George Parker Bidder*[1] (1806-1878), "the elder Bidder," was the son of a stone-mason of Devonshire, England. The indications of hereditary influence are stronger in the Bidder family than in that of any other calculator. Bidder's eldest brother, a Unitarian minister, had an extraordinary memory for Biblical texts, but no special arithmetical gift; another brother was an excellent mathematician and an insurance actuary; a nephew early showed remarkable mechanical ability; Bidder's eldest son, George Parker Bidder, Jr. (hereafter referred to as "the younger Bidder"), inherited in considerable degree his father's gift for mental arithmetic, together with his uncle's mathematical ability, being seventh wrangler at Cambridge in 1858; and two daughters of the younger Bidder showed "more than average, but not extraordinary powers of doing mental arithmetic." [2] Other members of the family were distinguished in non-mathematical ways.

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[1] Scripture, *op. cit.*, p. 23; *Proceedings Institution of Civil Engineers*, XV, 1855-6, p. 251; LVII, 1878-9, p. 294; Colburn's *Memoir*, p. 175; *Phil. Mag.*, XLVII, 1816, p. 314; *Spectator*, LI, 1878, pp. 1634-5; LII, 1879, pp. 47, 111.

[2] *Spectator*, LI, 1878, pp. 1634-5. In this article the younger Bidder is referred to as Mr. G. Bidder; but his full name was the same as that of his father, George Parker Bidder. (*Cf.* Jos. Foster's *Men-at-the-Bar*, 2nd ed., London, 1885, and *The Law List*, London, for 1882.) Scripture refers to both father and son, in different places, as George Bidder, and to the son usually as George Bidder, Q. C., Mr. Bidder, Q. C., or the younger Bidder; by Bidder (unqualified) he always means the elder Bidder, except in one case (p. 28), where the context prevents any misunderstanding. After noting that the similarity of the two names has caused some confusion, he tell us (*loc. cit.*), somewhat dogmatically, that "the only way out of the difficulty is to distinguish the son by adding his title [Q. C.]." (Why would not the son's A. B., or A. M., or his date of birth, or the father's C. E., answer just as well?)

Despite this device for avoiding difficulty, Scripture has fallen into sad confusion in dealing with the various members of the Bidder family. On p. 28 of his article he quotes from the *Spectator* (*loc. cit.*) the sentence: "If I perform a sum mentally, it always proceeds in a visible form in my mind; indeed, I can conceive no other way possible of doing mental arithmetic", omitting the comma after "mentally", but

At the age of 6 Bidder learned from an elder brother to count to 10, then to 100; this was the only formal instruction in figures he ever received. From counting by units to counting by 10's, and then by 5's, was a natural development. He then set about learning the multiplication table up to 10 x 10, with the aid of shot, marbles, etc., until, as he expresses it, the numbers up to 100 became his friends, and he knew all their relations and acquaintances. A year or so later his readiness in solving simple problems mentioned in his hearing attracted attention, and he acquired a considerable local reputation. Bits of mathematical information (such as that 10 x 100 means 1000, etc.) and halfpence contributed by his admirers conduced to the gradual development of his talent, aided by his natural keenness in reasoning about numerical relations; so that he was soon able to perform 4-, 5-, and 6-figure multiplications mentally. Meantime he came to observe various interesting properties of numbers,—the formulas for the sums of numerous series, casting out the 9's, short cuts in multiplication, properties of squares and of 2-figure endings, and the like. As a

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correctly attributing the remark to the younger Bidder. On p. 57, however, he makes the same quotation, this time adding a superfluous "of" after "conceive" and omitting the comma as before, but now attributing the quotation simply to *Bidder* (unqualified), meaning the *elder* Bidder, as the context unmistakably shows; for a little farther on he says, "This faculty was also inherited [transmitted?], but with a very remarkable difference. The *younger* Bidder [italics mine] thinks of each number in its own definite place in a number-form," etc.

But a worse confusion than this is still to be noted. The *Spectator* correspondence above cited, printed just after the elder Bidder's death, moved another correspondent (*Spectator*, LII, 1879, p. 143) to quote from *Brierley's Journal* for Jan. 25, 1879, the case of an eighteenth century Dissenting minister, the Rev. Thomas Threlkeld, who had a memory for Biblical texts similar to that of the elder Bidder's brother. On the strength of this, Scripture tells us (p. 27): "One of his [the elder Bidder's] brothers was an excellent mathematician and an actuary of the Royal Exchange Life Assurance Office. Rev. Thomas Threlkeld, an elder brother [!], was a Unitarian minister. He was not remarkable as an arithmetician, but he possessed the Bidder memory and showed the Bidder inclination for figures, but lacked the power of rapid calculation. He could quote almost any text in the Bible, and give chapter and verse. [Here Scripture gives the correct reference for this last sentence, which is taken from the younger Bidder's letter, and refers to the brother of the elder Bidder.] He had long collected all the dates he could, not only of historical persons, but of everybody; to know when a person was born or married was a source of gratification to him." Here we are given the correct reference for this last sentence, which refers to the Rev. Thomas Threlkeld, and is from the later volume of the *Spectator*. Thus by a piece of carelessness, hard to excuse, Scripture has inextricably confused the brother of the elder Bidder with this Rev. Thomas Threlkeld, who, so far as we know, was related to the Bidder family only by common descent from Adam!

result of this "natural" algebra and number-theory he hit upon some ingenious methods of performing complex operations; in particular, by his 11th year he was already in possession of a method by which he could solve compound interest problems mentally in an amazingly short time, in fact, almost as rapidly as a good computer using a table of logarithms.[1] Later, after his meeting and competitive test with Zerah Colburn, in 1818, he acquired great skill in the extraction of roots and the finding of factors, by methods similar to Colburn's, but with improvements of his own.[2]

Bidder's reputation soon became more than local, and when about 8 years old he was exhibited in various places by his father, after the fashion so recently set by the Colburns. But Bidder's admirers, more energetic than Colburn's, actually raised a fund to pay for his education, and put him in school. Later on, when his father resumed the profitable exhibitions, friends once more intervened, this time with permanent success. The boy was placed with a private tutor, and in 1819 attended classes in the University of Edinburgh, where he took a mathematical prize in 1822. Leaving the university in 1824, he held positions successively in the Ordnance Survey and in an assurance office. But by the advice of his friends he later decided to devote himself to civil engineering, and ultimately became one of the most successful engineers of his time. He was connected with several engineering undertakings of the first magnitude, and as a member of the Institution of Civil Engineers took a prominent part in the controversies then

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[1] On the mathematical side, if $P$ represents the principal, $r$ the interest (as a fraction of the principal, not as a per cent.), and $n$ the number of years, Bidder's method amounted to the expansion of the expression $P(1+r)^n$, by the binomial theorem, to a sufficient number of terms to insure accuracy in the last farthing. The properties of several numerical series were skillfully utilized at different stages of the expansion. (*Cf. Proc. Inst. C. E.*, XV, p. 267, for Bidder's own account.)

[2] Colburn says of this meeting (*Memoir*, p. 175), "Some time in 1818, Zerah was invited to a certain place, where he found a number of persons questioning the Devonshire boy. He [Bidder] displayed great strength and power of mind in the higher branches of arithmetic; he could answer some questions that the American would not like to undertake; but he was unable to extract the roots, and find the factors of numbers." Thus it would seem that Bidder's mind was not strongly turned in the direction of this class of problems until after this meeting with Colburn, but that once he became interested in them he soon outstripped his rival. Strangely enough Scripture, after mentioning this passage from the *Memoir* in his general bibliography on Bidder, does not cite it in his account of the meeting of Colburn and Bidder, but refers only to the one-sided account of a London paper, which represents Bidder's triumph as complete. For a further discussion of this meeting, see Appendix I.

before the profession. Constant use kept up his calculating powers, and in various railway and other contests before Parliamentary committees his great command of statistics and keen powers of analysis made him a formidable witness.

It would seem that Bidder's powers of mental calculation increased steadily at least up to the beginning of his university days, if not later,[1] and thereafter remained almost undiminished to the end of his life. Both in numerical calculations and in his study of higher mathematics he was interested in general principles, practical applications, and striking properties, rather than in intricate analysis for its own sake, or calculations with numbers chosen merely for their length. At Edinburgh he maintained a good class standing in mathematics, including differential and integral calculus, but only by hard study.[2] In the solution of problems where special properties or symmetries played a part he was equalled, if at all, only by such great calculator-mathematicians as Gauss and Ampère. In division his skill was considerable. In multiplication he was able, on one occasion, to handle two 12-figure numbers, but only by "a great and distressing effort";[3] in general, he

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[1] In the *Spectator*, LII, 1879, pp. 111-112, are given specimens of Bidder's feats during the years 1816-1819, with times of solution, also the London newspaper account of his meeting with Colburn to which reference has already been made. Scripture (*op. cit.*, p. 26) argues from this series that Bidder's powers increased between 1816 and 1819. That this was the case can hardly be doubted; but it certainly is not proved *by this series of examples*. Even comparative times for an expert computer solving these same problems on paper would prove nothing, since in several cases there are two or three different ways of doing the work, and possible short cuts which it is impossible to say whether Bidder noticed or not. Moreover, no two of the problems are alike. Perhaps the hardest problem of the lot is the compound interest question (1816, solved in 2 minutes) which is first in the list. The cube root of the 18-figure number (1819, 2 minutes) is far easier than it looks; for by this time, a year after his meeting with Colburn, Bidder was doubtless familiar with the application of 2-figure endings to these problems, so that he had only to find the cube root of the first 9 figures by trial and approximation to get the first three figures of the root, then add on the last two by inspection from the last 2 figures of the given number, and find the missing 4th figure of the root by casting out the 9's. The algebraic problem which was solved "instantly" in 1819 was very simple, and was undoubtedly solved by inspection; the answer, 3, was, from the nature of the question, the most natural first trial, and hence no special credit belongs to this last feat. These considerations show how difficult it is to reach definite conclusions from particular problems of this sort unless there is at hand specific knowledge of the detailed methods and short-cuts actually used in the examples under consideration, particularly of any special peculiarities of the given numbers whereby the solution may be facilitated.

[2] *Proc. Inst. C. E.*, XV, p. 253; *Spectator*, LI, 1878, pp. 1634-5.

[3] *Proc. Inst. C. E.*, XV, p. 259. In view of this explicit statement

did not cultivate his calculating power much beyond the limits of its practical usefulness to him. In his lecture "On Mental Calculation," before the Institution of Civil Engineers,[1] to which reference has already been made, Bidder has left us an excellent account of his methods of calculation.

*George Parker Bidder, Jr.*[2] (b. 1837), "the younger Bidder," was the eldest son of G. P. Bidder. Practically the only information we have concerning his powers of calculation consists of a few facts brought out in the *Spectator* correspondence already referred to. He was 7th wrangler of his year, and later a thriving barrister and Queen's Counsel. He tells us that he was unable to approach his father in extent of memory and rapidity and accuracy of calculation; we have seen, however, that the father, writing in his 50th year (after which his powers can hardly have shown any considerable increase), speaks only of multiplying 12 figures by 12 figures "on one occasion", by "a great and distressing effort", whereas the son was able, in several instances, to perform 15-figure multiplications, though slowly and with occasional errors. That the younger Bidder's method of multiplication was, like Diamandi's, cross-multiplication, we may infer from the fact that he incorrectly attributed this method to his father. Of the son's other feats in calculation, and of the degree of his precocity in this field, we have no knowledge. He was of visual memory type, and possessed a number-form running from right to left, the numbers up to 12 being arranged in a circle as on a clock. He declared that his calculations "proceed in a visible form" in his mind, and that he "can conceive no other way possible of doing mental arithmetic," which, as Proctor points out,[3] is a rather strange remark. Unlike most of the other calculators, he employed a mnemonic system instead of natural memory in remembering numbers. He could play two games of chess blindfold simultaneously.

*Truman Henry Safford*[4] (1836-1901) was, like Zerah Colburn, the son of a Vermont farmer; but both his parents were

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from Bidder himself, his son and Elliot seem to be wrong in attributing to him (*Spectator*, 1878, p. 1634) great facility in 15-figure multiplications. The son's statement that his father used cross-multiplication is likewise at variance with the father's explicit account of his method of multiplication (*Proc.*, XV, p. 260).

[1] *Proceedings*, XV, pp. 251 f.

[2] Referred to by Scripture as George Bidder, Q. C. Scripture, *op. cit.*, p. 28; *Spectator*, LI, 1878, pp. 1634-5; Galton, *Inquiries into Human Faculty*, pp. 133-4, and Plate I, 20, opp. p. 380.

[3] *Belgravia*, XXXVIII, 1879, p. 461.

[4] Scripture, *op. cit.*, p. 29; Appleton's *Cyclo. of Am. Biog.*, art. *Safford*; *Chambers's Edinburgh Journal*, N. S. VIII, 1847, p. 265; *Belgravia*, XXXVIII, 1879, p. 456.

former school-teachers, and persons of some education. The father had a strong interest in mathematics, and the mother, we are told, was of an "exquisite nervous temperament." Young Safford showed a remarkable all-round precocity, similar to that of Ampère. In his 3rd year "the grand bias of his mind was suspected"; later his parents "amused themselves with his power of calculating numbers"; and when he was 6 years old he was able to calculate mentally the number of barleycorns, 617,760, in 1040 rods. At the age of 7 he had "gone to the extent of the famous Zerah Colburn's powers." About this time he began to study books on algebra and geometry, and soon afterwards higher mathematics and astronomy. Wanting some logarithms, he found them himself by the formulas; and in his 10th year he published an almanac computed entirely by himself. The following year he published four almanacs, one of which, computed for Cincinnati, at once reached a sale of 24,000 copies. In this almanac he used a new and original rule for obtaining moon risings and settings, accompanied by a table which saved a quarter of the work of their computation. About this time he also discovered a new rule for calculating eclipses, with a saving of one-third in the labor of computing.

Such feats at once made the boy a public character, and in the same year (1846) he was examined by the Rev. H. W. Adams, a skillful mathematician. He solved a number of difficult algebraic problems, doubtless in the main by algebraic methods rather than by the trial and error method of most of the other prodigies. Problems in the mensuration of solids caused him no trouble, though in one case, where the answer was a 12-figure number, he "used a few [written] figures." He extracted the cube roots of 7-figure exact cubes "instantly," doubtless by the use of 2-figure endings. Finally, he squared 365,365,365,365,365,365, entirely in his head, in "not more than one minute,"[1] though with evident effort. A three-hour examination convinced Adams that the boy had mastered and gone beyond all his text-books.

Like Ampère, Safford had a wide range of interests, and an encyclopedic memory. Chemistry, botany, philosophy, geography, and history, as well as mathematics and astronomy,

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[1] All these quotations are from the *Chambers's Journal* article cited above. The last problem is there given as 365,365,365,365,365 x 365, 365,365,365,365,365, *i. e.*, a 15-figure number multiplied by an 18-figure number; but since the answer contains 36 figures, it is obvious that another 365 is omitted from the first number, and that the problem was the squaring of an 18-figure number. The repetition of the same figures, however, greatly simplified the work, there being only three different partial products. Scripture carries over the typographical error without comment, evidently not noticing it.

were included in his field of study. He took his degree at Harvard in 1854, and became an astronomer. After holding various positions he became professor of astronomy in Williams College in 1876, where he remained until his death in 1901.

Safford early outstripped Bidder in range of mental calculation, but with the aid of books, whereas Bidder's methods were entirely of his own discovery. It is to be regretted that we have not more detailed information about Safford's calculations;[1] but except for the examination whose results have been given above, all we can say is that later he acquired considerable skill in factoring large numbers, seeming to be able to recognize almost at a glance what numbers were likely to divide any given number, and remembering the divisors of any number he had once examined.[2]

*André Marie Ampère*[3] (1775-1836), like his successor Safford, showed all-round precocity, a wide range of interests, and

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[1] The *Chambers's Journal* article is written in rather florid style, and in a tone of admiration almost verging on awe. The Rev. H. W. Adams, who is there said to have been a skillful mathematician, was by no means as critical an examiner as might be wished. Thus while he tells us that several of the problems given were among the hardest in Davies' *Algebra*, he later notes that Safford already owned this work and had fully mastered it, hence had seen all these problems before. The times indicate, to be sure, that Safford calculated the answers afresh; but the test is not as satisfactory as if the problems had been entirely new to him. The times given, too, are mostly in the form "about a minute," and in definiteness leave much to be desired. The big number selected for the grand final test was about as unsuitable for the purpose as any that could well have been chosen. Not only in the recurrence of the same three partial products, but in the repetition of the same group of figures within each partial product, the problem is so artificially simple that it proves almost nothing concerning Safford's power of multiplication. The number 365, too, owing to its connection with the calendar, is especially easy to remember. Adams speaks of the "long and blind sums" which Safford remembered after a single hearing; but apart from this simple 18-figure number (which would not overtax the memory of any child who could keep in mind 365 and count six), the longest numbers in the statement of any of the problems mentioned in the article were of 7 figures. Now a normal boy of 13 can, on the average, retain 8.8 figures after a single hearing, and a boy of 11, 6.5 figures. Hence, while Safford's memory for figures was probably above the average, the fact is not satisfactorily proved by Adams' examination. (*Cf. American Journal of Psychology*, II, 1889, p. 607. The figures are erroneously quoted by Scripture, p. 41, as 8.6 for boys of 19 years, instead of 8.8 for boys of 13 years.) The fact that the *answers* to some of the problems were longer numbers is not relevant here; for, as we shall see later, there is an important distinction between figure-memory as such, and memory as it stands *in the service of calculation*.

[2] *Belgravia*, XXXVIII, 1879, p. 456.

[3] Scripture, *op. cit.*, p. 6; Arago's Éloge d'Ampère, tr. in *Smithsonian Report*, I, 1872, p. 111. The writer has been unable to consult the other references which Scripture cites.

an omnivorous memory. He learned counting at the age of 3 or 4, by means of pebbles, and was so fond of this diversion that he used for purposes of calculation pieces of a biscuit given him after three days' strict diet. He became a noted mathematician, and was also prominent in several other directions. Of his mental calculations, however, we have no specific information; his later achievements so overshadowed his early gift that his biographers are silent about it, and his case sheds little light on the problems connected with the subject.

*Carl Friedrich Gauss*[1] (1777-1855) was the son of a poor family; a maternal uncle of his, however, was a man of considerable mathematical and mechanical talent. When not quite 3 years old, Gauss, according to an anecdote told by himself, followed mentally a calculation of his father's relative to the wages of some of his workmen, and detected a mistake in the amount. Entering the gymnasium at the age of 11, he mastered the classical languages with incredible rapidity. In mathematics he was not only head of the class, but soon outstripped his teachers. At the age of 10 he was ready to begin the study of higher analysis, and at 14 he could read the works of Euler, Lagrange, and Newton. He became one of the foremost mathematicians of his time. His *Disquisitiones Arithmeticæ*, published at the age of 24, is practically the foundation of the modern theory of numbers.

Concerning Gauss' mental calculations we have for the most part only general information. His power seems to have lasted all his life, and to have exceeded that of any other calculator except Dase. He had a "peculiar sense for the quick apprehension of the most complicated relations of numbers," and "an unsurpassed memory for figures," and used from memory the first decimals of logarithms in his mental operations. He was especially fertile in inventing new artifices and methods of solution.

MINOR PRODIGIES.—In the following list are grouped a few calculators about whom too little is known for an extended account, but who present one or more points of interest.

*The Daughter of the Countess of Mansfield*[2] (b. about 1804)

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[1] The writer has followed Scripture's account of Gauss (*op. cit.*, p. 7), not having access to the sources there cited.

[2] Gall, *op. cit.*, V, p. 88; Colburn, *Memoir*, p. 174; Scripture, *op. cit.*, p. 32. The reference to the *Med. and Philos. Jl. and Rev.* given by Scripture can hardly be correct, since the young lady, being about Colburn's age, was in 1811 only 6 or 7 years old, and could hardly have had an American reputation. The exact words in Scripture's text are found in Gall's *Organology;* the *Jl. and Rev.* reference probably refers to Mr. Van R., of Utica. In fact, all the notes to page 32 of Scripture's article are incorrect except a few of those to Gall, where the absence of a page reference covers up the inaccuracy. The trou-

was seen by Spurzheim in London at the age of 13, at which time she "extracted with great facility the square and cube roots of numbers of nine places." Whether this refers only to *perfect* squares and cubes cannot be decided. Colburn speaks of her simply as displaying, in 1812, at the age of 8 or thereabouts, "a certain degree of mental quickness [in calculation] uncommon in her sex and years." Except for Bidder's two granddaughters, whose powers were but little above the average, she is the only girl calculator on record.

*Richard Whately*[1] (1787-1863) began to calculate at the age of 5, and retained the power for about three years; he probably surpassed Colburn, but did not happen to hit on Colburn's favorite problem of extracting square and cube roots. When he went to school the power left him, and at ciphering he was always "a perfect dunce."

*Mr. Van R., of Utica,*[2] like Whately, developed a gift for calculation at an early age (6 years), but lost it at the age of 8.

*Dr. Ferrol*[3] (b. 1864) has a sister about a year his elder, who shares his gift for mental calculation. His father was an architect and a good reckoner, and his mother's mind was occupied with architectural computations at the time of the birth of these two children; whether this prenatal influence had any effect on their mental powers cannot be determined. Ferrol's gift showed itself at an early age, but as soon as he learned the elements of algebra, at the age of 10, he developed a preference for mental algebra instead of mental arithmetic. He was head of his class in mental arithmetic, but below the average in all other studies. He is a remarkably poor visualizer. His processes are "intuitive"; the answer to a problem, he tells us, comes "instantly," and is always correct. His general memory is probably about normal; his figure memory depends on mnemonics.

*A blind Swiss* mentioned by Johannes Huber[4] not only solved

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ble seems to be due to a transposition; note [1] should be note [7] and all the others should be moved up a line, [2] becoming [1], etc. Colburn's account of the daughter of the Countess of Mansfield is quoted in full in Appendix I.

[1]Scripture, *op. cit.*, p. 10. The writer has been unable to consult the *Life* of Whately there cited. By an inadvertance, Scripture, on p. 57, gives the age of Whately's first calculations as 3, whereas, on p. 10, the statement is "between five and six."

[2]Gall, *op. cit.*, pp. 87-8, quoting *Med. and Philos. Jl. and Rev.*, III, N. Y., 1811. Gall mentions several other calculators, but it has not seemed worth while to enumerate them all here.

[3]P. J. Möbius, *Die Anlage zur Mathematik*, 1900, p. 73. The name is given simply as "Dr. Ferrol"; we are not told whether he is an M. D., or what are his initials.

[4]*Das Gedächtniss*, Munich, 1878, p. 43.

the most difficult problems, but could repeat a series of 150 figures either forwards or backwards after a single hearing, or name at once the 30th or 50th figure, *e. g.*, from either end. Before becoming blind he had been a man of very weak memory; but afterwards, busying himself with exercises in calculation, he discovered a very simple method of dealing with the largest numbers, and tried to sell his secret in England for a high price.[1]

*Vito Mangiamele*[2] (b. 1827), the son of a Sicilian shepherd, himself tended sheep, and when examined by the Academie des Sciences, at the age of 10, answered several questions, among them the cube root of 3,796,416 (=156), which he found in half a minute. Cauchy, in his Academie report on Mondeux, already cited, complains that Mangiamele's masters have always kept secret the boy's methods of calculation; it is not clear whether this means that they knew and refused to tell, or that the boy himself was unable to enlighten them. He was quite uneducated. A brother and a sister of his were also noted calculators.

*Prolongeau*[3] (b. about 1838), at the age of 6½, solved mentally with great facility problems relating to the ordinary operations of arithmetic, and to the solution of equations of the first degree.

*Grandmange*[4] (b. about 1836), born without arms or legs, performed, mentally, very complicated calculations and solved difficult problems.

*Mathieu le Coq*[5] (b. about 1656), an Italian boy, "at the age of 6, without knowing how to read or write, commenced to perform all the most difficult operations of arithmetic, such as the four elementary operations, the rule of three, partnership (*compagnie*), square and cube root, and that, too, as soon as the question was put to him." He learned to calculate by stringing beads.

*Vincenzo Succaro*[6] (b. 1822), a Sicilian, appeared in public as a calculator at the age of 6, received a good education, but showed no special mental ability outside of calculation.

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[1]Euler, it is well known, possessed considerable powers of mental calculation after becoming blind; but to what extent he had the power before his blindness, and just what feats he could perform, the writer has been unable to discover.

[2]*Comptes rendus hebdomadaires des séances de l'Académie des Sciences*, IV, 1837, p. 978; *Riv. sper. di Fren.*, XXIII, 1897, p. 434.

[3]*C. R. Acad. des Sci.*, XX., 1845, p. 1629.

[4]*Ibid.*, XXXIV, 1852, p. 371.

[5]Binet, *op. cit.*, p. 3; *Riv. sper. di Fren.*, XXIII, 1897, p. 430.

[6]The source for the remaining calculators is the *Riv. sper. di Fren.*, XXIII, 1897, pp. 429 f. A summary of this article in German is found in the *Zeits. f. Psy. u. Physiol. d. Sinnesorgane*, XVI, 1898, pp. 317-8.

*Giuseppe Pugliese* (b. "a little later"), also a Sicilian, took to the stage at the age of 5, and was exhibited in Italy and Germany. An attempt was made to teach him Geometry; but he was unable to deal with geometrical forms.

*Luigi Pierini* (b. 1878) learned late to speak and to walk, suffered from many children's diseases, and was an epileptic. He tended sheep, and thus learned to count. He developed a remarkable talent for mental arithmetic, and at an early age became a professional calculator.

II.

The writer will now give an account of his own case, which differs in three respects from those hitherto considered:

(1) The power is almost confined to dealing with the last two figures, or 2-figure endings, of the numbers used. It is readily seen that, with certain limitations in division and evolution, the last two (or $n$) figures of the numbers used in a given problem determine the last two (or $n$) figures of the answer, no matter what the preceding figures may be. Now the writer's mental calculations take the form almost exclusively of tracing the last two figures through the different operations, ignoring all the other figures. This evidently simplifies the work immensely.

(2) By a further specialization, the problems which he solves most often and most readily are of the general form of finding the last two figures of any power (or integer root) of any number.

(3) Finally, he has a strongly marked preference for working with even numbers. By a special method, to be explained later, he practically always changes odd numbers into even numbers for purposes of calculation, where only the last two figures of the answer are required; the even number thus obtained is readily converted into the desired odd number by very simple rules.

It will thus be seen that the writer's calculations are highly specialized, and in extent perhaps not comparable to those of any calculator heretofore considered. At the same time, some of these specializations are found in other calculators; and in the general features of its development the writer's case is typical of many or most of the others, and will, it is hoped, throw light on several points which have hitherto not been fully understood. While many of the details are in themselves of little importance, they will serve to illustrate the sort of numerical properties which not only facilitate mental calculation, but arouse the interest of the calculator, and hence furnish the motive for continued practice until the calculating habit becomes firmly established.

In the matter of heredity the only circumstance that need be mentioned is that the writer's younger brother has shown rather more than average ability as a chess-player, and has, on a few occasions, played a game blindfold; but by what psychological processes, or to what extent the power could be increased by further practice, cannot be stated. Nor need the writer speak of his school and college work, except to say that while he has always been fond of mathematics, it has no better claim than two or three other subjects to be called his favorite study.

His interest in mental calculation dates from the time he learned to count, at the age of 4, or possibly 3.[1] He learned to count to 10, then to 100, then beyond, and also to count by 2's, 3's, etc. Now in these latter series 2x2, 2x2x2, 3x3, 3x3x3, etc., in short, the powers of the number by which he was counting, were natural resting-places, and awakened his interest, so that before long he began to count in the power series of different numbers (2, 4, 8, 16, 32, etc., 3, 9, 27, 81, etc.) for considerable distances. At first he simply emphasized the powers as they occurred in the complete series of multiples, but gradually he learned to omit the intermediate multiples, and simply count in the power series proper: 2, 4, 8, 16, etc., 3, 9, 27, 81, etc. But almost always, when the number exceeded 100, he emphasized the last two figures, and gradually got into the habit of ignoring all the others. Thus instead of saying 3, 9, 27, 81, 243, 729, 2187, etc., he usually counted 3, 9, 27, 81, 43, 29, 87, and in this simplified form counted along the different power series for considerable distances. Multiplication naturally grew out of this counting process; but it was really *counting* rather than multiplication proper, since he did not learn the multiplication table until some time later, when he went to school. Thus to find 9x7 at this time he would count 9, 18, etc., to 63; and even now, except within the limits of the multiplication table as he learned it to 12x12, his mental multiplications are abbreviated countings of this sort (skipping most of the intermediate links) rather than true multiplications. We have already seen reason to suspect that neither Buxton nor Tom Fuller really got beyond this counting process into true multiplication, *i. e.*, with the use of a memorized multiplication table.

In the course of these calculations or countings, a number of properties gradually attracted the writer's attention; such as that every power of a number ending with 0 or 5 ends with 0

[1] Unfortunately, definite dates cannot be given. The power developed very slowly, never really becoming important for any but psychological purposes, so that no one but the writer himself knew of its existence until a much later date.

or 5, that the 4th power of any other number ends with 1 or 6, according as it is odd or even, that the 5th power ends with the same figure as the 1st, the 6th with the same figure as the 2nd, etc.; and that if 76, or any number ending with 76, is multiplied by a multiple of 4, the last two figures of the product are the same as those of the multiplier (*e. g.*, 76x12=912). Then he noticed that the ending[1] 76 occurs at various points in the power series of different numbers (the 5th power of 6, the 4th power of 32, the 2nd power of 24, the 10th power of 4, the 20th power of 2, etc.,), and that from these points the series of endings repeats, except that in some cases the ending of the next power will differ by 50 from that of the original number. Thus the endings of the first 20 powers of 2 are 02, 04, 08, 16, 32, 64, 28, 56, 12, 24, 48, 96, 92, 84, 68, 36, 72, 44, 88, 76; the 21st is 52 instead of 02; but the 22nd is 04, like the 2nd, and thereafter the endings recur in regular order. Finally it turned out that the *20th* power of every even number (not ending with 0) had the ending 76, and that odd numbers had a similar property, the 20th power ending being, however, 01 instead of 76, and even the 21st power being always the same as the 1st, except for multiples of 5.

After discovering these and similar properties, the writer found it a simple matter to find the last two figures of any power of any number, by counting along the proper series. The process was always, however, of the counting type already indicated. Thus to find the 8th power of 3 the process would be 3, $^{6}$, 9, $^{18}$, 27, $^{54}$, 81, $^{62}$, 43, $^{86}$, 29, $^{58}$, 87, $^{74}$, 61; *i. e.*, he would count up to a power of 3, then by this power to the next, and so on, but passing very lightly over the intervening multiples, and in time learning to omit them altogether. In fact, before long the process came to be simply, *3*, 9, 81, *61*, *i. e.*, simply squaring each number to get the next, the intermediate countings taking place so rapidly and automatically as hardly to appear in consciousness at all, except as brief "flashes." And even these "flashes" may sometimes be almost absent, so that only the 3 and the 61 stand out, the rest remaining a mere blur.

It happened that about the time he learned to count, and for perhaps two or three years thereafter, the writer was frequently ill. This, of course, left a large amount of time free for his calculating exercises, and probably had not a little to do with strengthening his bent in that direction.

We come now to the third peculiarity mentioned above: the writer's preference for even numbers. An examination of the

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[1] The term "ending" (unqualified) will hereafter be taken as synonymous with "2-figure ending."

following table of the endings of certain products will form the best introduction to this subject.

| | 07 | 32 | 57 | 82 |
|---|---|---|---|---|
| 23 | 61 | 36 | 11 | 86 |
| 48 | 36 | 36 | 36 | 36 |
| 73 | 11 | 36 | 61 | 86 |
| 98 | 86 | 36 | 86 | 36 |

It will be observed that the numbers at the left, 23, 48, 73, 98, differ in pairs by 25, and so with the numbers at the top, 07, 32, 57, 82; and that in each case there is one multiple of 4 (48, 32), one odd multiple of 2 (98, 82), one number of the form $4c+1$ (73, 57), and one number of the form $4c-1$ (23, 07). Now the 16 numbers in the body of the table, it will be seen, all belong to a similar series, 11, 36, 61, 86. If either of the factors is a multiple of 4, the product has the ending 36, as shown by the 2nd line and the 2nd column; if both are odd multiples of 2 (98, 82), the product again ends with 36; if one is an odd multiple of 2 (98, 82), and the other an odd number, the product has the ending 86, $=36+50$. Finally, if both numbers are odd, the ending of the product is $36\pm25$, *i. e.*, either 11 or 61:- 61 (a number of the form $4c+1$) if the numbers multiplied are either both of the form $4c+1$ ($73\times57$), or both of the form $4c-1$ ($23\times07$); and 11 (a number of the form $4c-1$) if one of the factors is of the form $4c+1$ and the other of the form $4c-1$ ($73\times07$, $23\times57$). Thus by applying a few simple rules, any one of the 16 products in the table can be made to depend on the single product, $48\times32$, of the two multiples of 4 in the table. Hence to find the ending of the product of two odd numbers, change each into a multiple of 4 by adding or subtracting 25, multiply these multiples of 4 together, and then add or subtract 25, as the case may require, to get the answer. A similar principle obviously applies to the power series of any odd number; simply find the required power of the corresponding even number, and then either add or subtract 25.

Now these properties early attracted the writer's attention, and he soon got into the habit of transforming odd numbers into even numbers in practically all his calculations. The result was that (if we leave out of account multiples of 5, which belong to a class by themselves and are very easy to multiply) the whole of multiplication, so far as the endings were concerned, was reduced to the 200 possible products of any two of the 20 numbers 04, 08, 12, 16, 24, 28, 32, 36, etc.; whereas in order to do the same work without this transformation, the 3200 combinations of the whole eighty 2-figure end-

ings prime to 5 would have to be considered. In finding powers, again, he had to deal with only 20 different series, each of which repeated after 20 terms or less; so that the whole problem of finding the last two figures of any power of any number was reduced to less than 400 simple cases, instead of an indefinite number of cases. He never committed these products and powers to memory; it was not necessary; with practice he was soon able to count to any desired one with great rapidity, in fact, just as rapidly, in the simpler cases, as he could have recalled the answer if it had been previously memorized.

To recapitulate: The writer's mental calculations usually deal only with the 2-figure endings of numbers, rejecting all previous figures if there are any; by far the commonest problem is to find (the ending of) some given power of a given number, or to investigate some property of some power or group of powers of one or more numbers; and problems involving odd numbers (except, of course, odd exponents) are almost always solved by changing the odd numbers into multiples of 4 (by adding or subtracting 25), and changing back to an odd number in the same way, if necessary, after the work of calculation is over. He might go on and indicate many other properties of numbers, or rather of endings, which he discovered and used in calculating; but enough has already been said to give a fair idea of the general nature of the processes employed, the gradual development of the calculating power, and the advantages of the various specializations which came to be adopted.

Of course his calculations are not absolutely confined within these limits. Besides finding endings in the power series of even numbers, he can also multiply endings very readily, and add or subtract them (by counting forwards or backwards) somewhat less rapidly, or divide them where the division is known to be exact; and he *can* work, though very much more slowly, with odd numbers. But even in the power series of 3, the odd series with which he has worked oftenest, it is easier in most cases to change 3 into 28; and in any other odd series he can scarcely work at all, except with the greatest effort. The even series in these other cases are so much easier and more familiar that it is practically impossible to resist the temptation to work in them, even when he tries to work laboriously in the odd ones as such.

When the calculation takes account of *all* figures of the result, not merely of the last two, the writer's powers of mental arithmetic are probably very little above the average, certainly not equal to those of any one who has had a moderate amount of practice in the work. Even the multiplication of two 2-fig-

ure numbers takes him longer mentally than on paper; and with 3-figure numbers it is such an effort for him to remember the partial products that usually each one must be repeated aloud two or three times, and even then he is apt to forget the first partial product by the time he has found the third. With small 2-figure numbers, however, he finds no difficulty in multiplying (on paper, using only one figure of the *multiplicand* at a time,) in a single operation, especially where the number is even, *e. g.*, 24 or 36. With 19 or 23, too, it would probably be easier for him to multiply in a single operation than in two operations in the ordinary way; but in such a case, after the products exceeded 100, the multiplication would often tend to resolve itself into counting,—rapid and automatic, but counting nevertheless. Thus up to $23 \times 5 = 115$ he would probably count by 23 directly, or depend on his memory; but after that, to pass to $23 \times 6 = 138$, he would first count in the 3, then the 20, thus reaching 138 from 115 *via* 118 and 128.

There are two cases in which the writer can find complete products with fair readiness. The first is in squaring numbers; here, however, the process is usually neither counting nor multiplication directly, but an application of some algebraic formula. Up to perhaps 32, and in certain other cases, such as 36, 48, 54, 64, 72, 81, 96, 144 (*i. e.*, numbers containing no other prime factors than 2 and 3), he would give the squares from memory; but usually he finds only the last two figures by memory, and gets the rest by interpolation between two known squares or by the formula for $(a+b)^2$.

The second case is where two numbers are to be multiplied, neither of which contains any prime factors except 2 and 3. Here his method is to count (multiply) by 2's or 3's to some convenient multiple of one of the numbers, then by that multiple to some other, and so on, until the required product is reached. Thus to find $48 \times 64$ he would count by 48 to 384 $(= 48 \times 8)$, then by 384 to 1536, then to 3072 $(= 384 \times 8 = 48 \times 64)$, the required answer. To square 162, again, the stages would be 486, 1458, 2916, 8748, 26244, *i. e.*, multiplying successively by 3, 3, 2, 3, 3.[1] In these cases much of the work would be automatic and half-unconscious. Thus up to 2916 $(= 162 \times 18 = 54^2)$ in the second example the numbers in full would be very familiar, and perhaps only the 58 of 1458 would

[1] Buxton, it will be remembered, in multiplying 456 by 378, multiplied successively by 5, 20, and 3, to get $300 \times 456$; then multiplied $456 \times 5$ and that product by 15, and added the result to $300 \times 456$, to get $375 \times 456$; and finally completed the operation by adding $3 \times 456$. This indicates pretty clearly that his method was like the one described above, a counting in the series of multiples of the multiplicand, rather than the ordinary method.

be distinctly formulated; but above that he would have to formulate all the figures distinctly and take account of them in the counting process. In the first example, 48 x 64, perhaps only the 84 of 384 and the 36 of 1536 would be distinctly formulated until the end, when 3072 would be given as a whole. It will be seen, then, that part or all of the intermediate numbers of the calculation may remain below the level of clear consciousness, and that where the numbers are familiar, part of a number may be in clear consciousness and not the rest of the number. At the same time the whole number functions in the calculation, otherwise the correct answer would not result.

There is just one class of problems in which the writer could compete with the real mathematical prodigies, *viz.*, finding the square and cube roots of exact squares and cubes. In fact, extracting the roots of perfect powers and testing the possible factors of given numbers are the only fields in which the properties of 2-figure endings are really useful, and even these problems, however interesting to the mathematical prodigy, are of little practical importance to the mathematician. Bidder, Colburn, and Safford made a specialty of these problems, and there is good evidence that all three solved them by the aid of the properties of 2-figure endings. A brief description of the "method of endings" will therefore not be out of place.

Given the last two figures of a number, the last two figures of its square are known; but given only the last two figures of a perfect square, the last two figures of the square root are not definitely known, although the possible values are usually only four in number. Similarly, an odd ending has only one possible cube root, but an even ending has either none, or two which differ from each other by 50. Now, suppose a given number is known or suspected to be a perfect square or cube, and its root contains only three figures. The first figure can readily be determined by inspection; and the last two figures must be one of a limited number of possible roots of the ending of the given number. It is usually easy, after a little practice, to tell almost at a glance which of the possible roots to choose in a given case. In doubtful cases (multiples of 5, *e. g.*, where the number of possible roots is greater) such expedients as casting out the 9's, squaring or cubing one of the suspected answers or some number near it, or using the 3-figure instead of the 2-figure ending, will help to decide which is the correct root.

The application to factoring is still simpler. If the number to be factored is not already odd and prime to 3 and 5, it is easily made so by simple division. Now, in the case of an odd number prime to 5, if the last two figures of one of its

divisors are known, and the division is exact, the last two figures of the other can have only one value; and it is easy to construct a table showing the different pairs of endings in the factors which will produce a given ending in the product. Now, suppose it is suspected that a given number is a factor. From the table, or by a computation in accordance with simple rules, which need not be considered here, find the last two figures of the other factor; if desired, the hundreds figure can also be determined by casting out the 9's. This done, it is necessary to carry the division only far enough to decide whether the required last two or last three figures can result; as soon as this is seen to be impossible, the work is abandoned, since only an exact divisor is wanted. It is thus evident that much work may be saved, especially where the numbers involved are not very large; indeed, a factor may often be rejected almost at a glance which would otherwise have to be divided through to the end.

So much for the application of 2-figure endings to evolution and to factoring. The latter problem never attracted the writer, owing to the habit he so early developed of confining his attention to the last two figures; but in any case where a given number is known to be a perfect square or cube, and its root contains not more than three figures, he finds no difficulty in discovering the root by inspection. This would apply almost equally to higher roots, except that in some cases it would be difficult to tell the root if it contained more than two figures; but in general, the higher the root the easier the problem, and square and cube roots are the only ones which often come up. It is evident, however, that skill in solving this class of problems does not imply special skill or quickness in other branches of mental arithmetic, and that a careful distinction must be made between the cases where the given number is a perfect power and those where it is not. Where the root is not an integer, the ending gives no aid in finding it; memorization of a large number of perfect squares and cubes, or some process of real calculation, must then be resorted to, instead of the simple method of guessing by inspection of the ending of the given number.

Before closing this part of the paper, the writer may say a few words about his memory type. He learned to count orally, and his calculations began at once, without further aid; he cannot remember ever counting on his fingers, using pebbles, or the like; and even when he learned to make written figures later on, they never came to be associated with his mental calculations, which remained strictly auditory (or auditory-motor) throughout. Ordinarily the motor element is almost entirely absent; when the calculations remain in the familiar fields

already described, they are accompanied by no perceptible innervation of the muscles of speech. When he attempts unpracticed feats, however, such as complete 3-figure multiplications, the tendency to pronounce some or all of the figures is marked.[1]

But while the writer's type is unquestionably auditory in calculation, the presence of written figures is not a hindrance to him, as it is to Inaudi. On the contrary, if the numbers involved are at all large,—say a 9-figure number whose cube root is to be found,—the presence of the number on a sheet of paper before him is a distinct aid, saving a considerable effort of memory, and greatly facilitating such tests as casting out the 9's. Outside of calculation the writer's type is predominantly auditory; but he can use visual images at will with no special difficulty, and in geometry or similar fields uses them habitually as a matter of course. In general, then, his type is mixed, but with a slight predominance of auditory images.

It only remains to add that his calculating powers have increased, though very gradually, from the time he learned to count until the present, constantly taking advantage of the results of his mathematical studies, and at intervals following out new lines of inquiry and classes of problems based upon new properties of numbers and endings. There has been no tendency, however, to enter the broader fields of calculation cultivated by the mathematical prodigies; in the main, his calculations are confined within the limits already described, and even within these limits it often happens that of two problems which, to an ordinary calculator, would be of equal difficulty, one will be far easier for him than the other, owing to the peculiar preferences which have guided the distribution of his practice in calculation. While mental arithmetic has never absorbed a disproportionate share of his time, there is scarcely a day in which some of the old familiar series do not at some odd moment or other run through his head, usually quite automatically. He has never had any fondness for written computation for its own sake, and is perhaps, if anything, a trifle slower at it than the average man with an equal knowledge of mathematics. He is liable to occasional errors unless he carefully tests every stage of his work.[2]

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[1]Much the same thing was true of Safford; we are told (*Chambers's Journal*, VIII, p. 265) that it was his custom to talk to himself when originating new rules, but, by implication, not when carrying on computations by familiar rules.

[2]In the foregoing account an attempt has been made for the most part to avoid technical terms that would not be clear to the non-mathematical reader. The student of the theory of numbers will readily recognize that "2-figure endings" are least positive residues

## III.

We are now ready to interpret the facts thus far set forth, and to construct from them an explanation of the mathematical prodigy.

*Heredity*. The table[1] gives such information as could be found concerning the heredity of the different calculators. Just what part these various circumstances actually played in the development of the different prodigies is a difficult question, which it would hardly be worth while to discuss here. This much is clear, however, that whatever the influence of heredity in some cases, it is in no sense an *explanation* of mental calculation, but at most a favoring circumstance. A satisfactory theory must rest on a much more definite basis than such general terms as heredity, environment, and the like can afford; it must explain the cases where hereditary influence is lacking, as well as those where such influence seems to be present. Hence we may safely leave the question of the relation of heredity to mental calculation for other investigators, and devote our attention to other questions.[2]

*Development*.—(*a*) *Precocity*. There is nothing more striking about the mathematical prodigies, nothing which has been the subject of more uncritical amazement, than their almost uniform precocity. Gauss began his calculations before he was 3 years old; the present writer, at 4; Ampère, between 3 and 5; Whately, at 5; Pugliese and Succaro, at about 5; Colburn, at 5: Safford, at 6 or earlier; Mathieu le Coq, Mr. Van R., of Utica, Bidder, Prolongeau, and Inaudi, at 6; Mondeux, at 7; the Countess of Mansfield's daughter, at 8 or earlier; Ferrol, Mangiamele, Grandmange, and Pierini, at early ages not definitely stated. Buxton's mental free beer record began from the age of 12; Zaneboni's calculations began at the same age; Dase attended school at the age of 2½, and took to the stage at 15. In short, precocity is unmistakably the rule; if we

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(modulus 100), and that the writer's process of changing odd numbers into even simply changes the modulus to 25 instead of 100, using residues which $\equiv 0 \pmod{4}$. It would be easy to generalize many of the properties described above, and to show their application to *n*-figure endings and congruences in general; but such a task would carry us far beyond the limits of the present paper.

[1] See Appendix II.

[2] There are two points, however, on which a word may be said. In the first place, it is a pretty safe guess that Colburn's extra fingers and toes were an accident, as far as his calculating power was concerned, and had no connection with his mental abnormalities. On the other hand, the nervousness which he showed, and which he shared with Safford and an unnamed calculator in the neighborhood of Troy, N. Y. (*Memoir*, p. 173), may have predisposed him to less active participation in childish games, if not to actual illness, and so have increased the time available for his mental calculations.

count as unprecocious Zaneboni, Buxton, Dase, Diamandi (who began to calculate at 16, but had excelled in mathematics in school from the age of 7), the slave Tom Fuller, and the younger Bidder (about whom nothing definite is known in this respect), we have at the worst 6 unprecocious calculators as against 18 who were precocious.[1]

To understand this precocity we must note, first of all, that arithmetic is the most independent and self-sufficient of all the sciences. Given a knowledge of how to *count*, and later a few definitions, as in Bidder's case, and any child of average ability can go on, once his interest is accidentally aroused, and construct, unaided, practically the whole science of arithmetic, no matter how much or how little he knows of other things. Addition is only a shortened form of counting. The same is true of multiplication;[2] the writer's own case shows that the calculator need not even sit down and teach himself the multiplication table, as Bidder did, but may multiply by simple modifications of his counting process. Involution is simply a modification of multiplication; it has already been pointed out that the powers of numbers are natural resting-places in counting along the series of multiples of the numbers. The inverse operations of division and evolution grow naturally out of the direct operations of multiplication and involution; much more easily and naturally in mental than in written arithmetic. Once these elementary operations are mastered, such processes as reduction of years to seconds, compound interest, and any

[1] From this list has been omitted Huber's blind Swiss, who learned to calculate, presumably, late in life, by artificial methods, and obviously does not belong to what Binet calls the "natural family of great calculators." Binet's average of 8 years (*op. cit.*, p. 191) for the precocious calculators is too high; it is obtained by rejecting (without sufficient ground, so far as the writer can see) the cases of Gauss (3 years old) and Whately (who, as we have seen, began to calculate at 5, not at 3 as stated inadvertently in Scripture's table), and by taking in several cases the age at which the prodigy was *exhibited* before the Acad. des Sciences as the age when his calculations began. But on Binet's own showing, Mondeux had calculated for three years before he was exhibited in Paris; so that it will not do to average together such dissimilar data. Where the age of exhibition is later than 7, no attempt has been made to date the beginning of the calculations; if we then average the ages of the known cases of precocity (some of which are undoubtedly too high by a year or more), we get 5 to 5½ as an average, not 8. This is much more natural if the "natural calculator" usually begins to calculate from the time he learns to count. Of the six men not known to be precocious, two (Fuller and Buxton) were densely ignorant, and two of the others belong to the visual type, which, as we shall see later, is in certain respects intermediate between the "natural" or auditory and the "artificial" type.

[2] In multiplication the counting is, of course, done in the series of *multiples* of the multiplicand, not in the series of natural numbers; *cf.* part II of the present paper.

other arithmetical problems are simply a matter of understanding the meaning of the question and then applying known rules, plus a varying amount of ingenuity, to the solution. In accordance with the tendency of all mental operations, psychological shortenings of the processes involved will come with practice, and mathematical properties of the sort already described still further facilitate the work; so that in favorable cases the whole process may become in large measure automatic, and may go on while active attention is given to something else.

Moreover, the various symmetries and properties of numbers and series attract the attention of the calculator from the start, and keep up his interest until the habit of mental calculation has been firmly fixed. After that, if nothing intervenes to change that interest, there is practically no limit to which he may not attain, as the case of Dase abundantly shows.

We must note, furthermore, that practically an unlimited amount of time may be available for these calculations if the prodigy wishes so to use it. Mental arithmetic requires no instruments or apparatus, no audible practice that might disturb other members of the family, no information save such chance scraps as may be picked up almost anywhere for the asking, or absorbed, without even the trouble of asking questions, from older brothers and sisters as they discuss their school lessons. The young calculator can carry on his researches in bed, at the table,—if he allows himself to be "seen and not heard,"—during the perhaps laborious process of dressing or undressing; in short, at almost any time during the twelve or fourteen hours of his waking day, except when he is engaged in conversation or active physical play.

Thus, if an interest in counting once takes hold of a child either not fond of play or not physically able to indulge in it,—and stringing beads, counting the ticks of a clock, or even a chance question like "Let's hear if you can count up to 100", may start such an interest, which will then furnish all the material for its own development,—he may go on almost indefinitely, and become a prodigy long before his parents suspect the fact. Indeed, the interest in counting may seem so natural to the child that he may never think of doubting that every one else possesses it, and months or even years may elapse before some accident reveals the direction of his interest to his astonished relatives. Several of the calculators—Mondeux, Mangiamele, Pierini, Inaudi—were shepherd-boys, an occupation which, since it requires an ability to count and affords ample leisure, is peculiarly favorable for practicing calculation; several, again,—Grandmange (born without arms or legs), Safford, Pierini, the present writer,—were sick or otherwise

incapacitated for active play to a greater or less extent, and thus enjoyed an equally good opportunity to practice calculation. Fuller and Buxton, on the other hand, whether precocious or not, were men of such limited intelligence that they could comprehend scarcely anything, either theoretical or practical, more complex than counting; and their purely manual occupations left their minds free to carry on almost without limit their slow and laborious calculations.

These considerations put the whole matter of mathematical precocity in a new light. Instead of joining in the popular admiration and awe of these youthful calculators,—and even psychologists have not been wholly free from this uncritical attitude,—we must say that precocity in calculation is one of the most natural things in the world. If a person is to become a calculator at all, he will usually begin as soon as he learns to count, and in most cases before he learns to read or write; and his development, while it will of course be gradual,—in Bidder's case probably a year elapsed between his learning to count and the early incidents which made his gift known,—will be so greatly facilitated by the amount of time available, the intrinsic interest of calculation, and the ease with which new information can be picked up as needed, that he may become a full-fledged calculator before he is suspected of being able to count without the aid of his fingers. His preoccupation with his calculations may give rise to a false appearance of backwardness, or he may really be of very low intelligence, or he may be an all-round prodigy like Safford, Gauss, and Ampère; mental arithmetic is so completely independent and self-sufficient that it is equally compatible with average endowments or with either extreme of intelligence or stupidity.

Mathematical precocity, then, stands in a class by itself, as a natural result of the simplicity and isolation of mental arithmetic. There is nothing wonderful or incredible about it. The all-round prodigy like Ampère or Sir William Rowan Hamilton or Macaulay is possible only in a well-to-do and cultured family, where books are at hand and general conditions are favorable, and he must possess genuine mental ability. The musical prodigy, again,—Mozart is the stock instance,—must come of a musical family, hear music, and have at least some chance to practice, and hence cannot long hide his light under a bushel. But the mathematical prodigy requires neither the mental ability and cultured surroundings of the one nor the external aids of the other. He may be an all-round prodigy as well, like Gauss, Ampère, and Safford; it is not improbable that Bidder, under favorable conditions, would have developed into such an "infant phenomenon"; but he may also come of the humblest family, and be unable, even under the most

favorable conditions, to develop average intelligence. He may proclaim himself to the world almost at once, like the all-round or the musical prodigy, or keep his gift a secret for months or even years. If we are to call him a prodigy at all, it is important to realize how widely he may differ from other prodigies, and to avoid carefully the popular confusion due to the misleading associations of the words "prodigy" and "precocious."

(*b*) *Loss of Power.* Mental calculation, then, starts from an interest in counting; at the outset it demands only that ability to count by 1's, 2's, 3's, 7's, and the like, which all of us require for such every-day purposes as keeping track of the days of the week. But if for any reason this interest in counting is lost, practice in calculation will cease, and the skill already acquired will disappear, just as the pianist's skill is lost when interest and practice cease. There are two striking instances of this among mental calculators: Whately and Mr. Van R. of Utica, both of whom began to calculate at an early age, but lost the power after two or three years. Here, again, however, there need be no mystery; the disappearance of the gift with the loss of the interest in which it originated is as natural and normal as its original appearance.

Just what caused the loss of interest is not always easy to say. In Whately's case the trouble may have been that on going to school he was taught arithmetic or "ciphering" by methods very different from his old ones, became confused, failed to establish a connection between the two, and lost his interest in calculation as a result of his distaste for "ciphering." In Colburn's case the loss of skill seems to have been much more gradual, and probably never complete. In this respect he is like the pianist who retains his interest in music, but is prevented by other occupations from keeping in practice; if later on he is able to resume practicing, his skill is soon regained.

*Education.* A glance at the table of mathematical prodigies[1] will show that education as such, whether mathematical or general, has little or no influence on the calculating power, either to help or to hinder it. At the one extreme we find Fuller and Buxton, men of dense ignorance and limited powers of calculation, and near them Dase, the greatest of all calculators, who even in mathematics was scarcely less stupid. At the other extreme stand Ampère, Gauss, Bidder, and Safford, in whom unusual mathematical and general ability and a wide range of interests exist side by side with marked skill in mental calculation; while, on the other hand, the ordinary

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See Appendix II.

mathematician or man of culture has little or no gift for mental arithmetic. That the calculating power should be independent of general education is not particularly surprising; but its independence of mathematical training and ability seems at first less natural and obvious.

In a general way, we may distinguish three grades of mathematical ability in the great calculators. Those of the first class never get beyond the stage of pure counting, though of course the counting process comes to be abbreviated more or less with practice. At this stage the point of view is not even arithmetical; the calculator thinks not of arithmetical operations, but of *properties* of numbers and of series, and the short-cuts he uses are of a relatively simple sort, showing no mathematical insight. Without insisting too sharply on the distinction, we may term these men "calculating prodigies."

Those of the second class may be called, from the present point of view, "arithmetical prodigies"; Colburn and Dase will serve as examples. Here we find a fairly well developed knowledge of arithmetic, and a distinctly arithmetical point of view; it is operations of calculation, rather than mere properties of numbers, in which these men are interested, and the various short-cuts used are, we may suppose, suggested by practice in calculation rather than by mathematical keenness.

The third class comprises the "mathematical prodigies"[1] proper, of whom Bidder may be taken as the type. Here we find real mathematical ability, power to take a distinctly algebraic point of view, to generalize, and hence to discover all sorts of ingenious short-cuts and symmetries. Bidder's compound interest method is perhaps the most striking example; Mondeux's unconscious use of the binomial theorem is another.

Such a classification must not be taken too seriously, of course; a good deal of hair-splitting would certainly be needed to establish hard and fast lines between the different classes. The important point is that mental calculation and mathematical ability are essentially independent, and that almost any degree of the latter is compatible with any degree of skill in the former. Where the two are found together, calculation usually appears first; but even to this there are exceptions,

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[1] These terms are here used merely as a convenient means of temporarily designating different grades of skill in calculation; the writer would not advocate their general adoption as here defined. For a general term embracing all three classes, "arithmetical prodigies" seems the best; the reason this term was not taken as the title of the present paper was that Scripture's article already bore that title, and it seemed undesirable to run the risk of confusing later students by adopting it for a second article on the subject.

since Diamandi excelled in mathematics at school for nine years before he discovered his gift for calculation.

Neither mathematical nor general education or mental ability, then, has any *direct* influence on mental calculation. Indirectly, however, education may have an important influence. We have seen that if for any reason the interest in calculation is lost, the calculating power will disappear. Now mental calculation is a narrow and special field, with little practical importance for most men; hence, other things being equal, as a boy's sphere of interests widens, his interest in mental calculation is likely to sink into the background. This explains why so many ignorant men have excelled as calculators; ignorance, by preventing the intrusion of other interests, leaves the calculator free to develop his one gift, and keeps him from realizing how trivial it is, and how groundless is the public amazement which, perhaps, contributes to his support. On the other hand, if the interest in calculation is retained despite the widening of the sphere of interests resulting from education, the calculating power may prove to be of considerable practical value. The two Bidders will serve as examples. The father owed his striking success as an engineer primarily to his powers of mental calculation, which not only won him the friends who contributed to pay the cost of his education, but were of constant use to him in his profession, especially as an expert witness before Parliamentary committees. The son, a lawyer, tells us that he finds it an immense advantage to have in mind a number of formulas and constants for ready reference,[1] and doubtless his readiness in using these formulas and constants in mental arithmetic was still more useful. Gauss and Safford are illustrations of the obvious possible usefulness of mental calculation to the mathematician.

*Calculation.* If mental calculation naturally arises out of counting, we might at first suppose that addition would be the favorite operation of the mathematical prodigies; but there is no evidence to this effect in any known case. Bidder specifically states that multiplication is the fundamental operation; Colburn found multiplication easier even than addition or subtraction; Buxton's favorite problems seem to have been long multiplications, yet we have seen reason to suspect that his calculations never progressed far beyond the counting stage; the younger Bidder performed 15-figure multiplications, Safford (though in a very easy case) an 18-figure multiplication, and Dase one of 100 figures which did not seem seriously to tax his powers. The reduction of years to seconds and similar problems, resting on simple applications of multiplication, have

[1] *Spectator*, LI, 1878, p. 1635.

been favorites with many calculators. In short, *multiplication* and not addition seems to be the fundamental and favorite operation in mental calculation.

Nor is this difficult to understand. It has been suggested above that in the earlier stages the "natural" calculator who begins with counting—as distinguished from the "artificial" calculator who begins relatively late in life, using book-methods from the start—is interested mainly in *properties* of numbers and of simple series. Now these properties are revealed not by addition, but by multiplication, or the forms of counting which are equivalent to multiplication. Addition and subtraction bring out no properties of particular interest. Given any number, another number can be added to it or subtracted from it so as to produce any other number whatever, by processes which are mechanical and not particularly interesting. Where the addition of a series of terms does produce a result of any interest, as in the series $1+2+3+4 \quad . \quad . \quad .$ , or the series $1+3+5+7 \quad . \quad . \quad .$ , the interesting property belongs to the sum not primarily as a sum, but as a function or *multiple* of the $n$th term: in the first case the sum of $n$ terms is $\frac{1}{2}n(n+1)$, which is a multiple of the next term $n+1$, or of half the next term in case that next term is even; while in the second case the sum of $n$ terms is always equal to $n^2$, which is the *product* of two equal factors. Addition and subtraction, in short, apart from multiplication, are mechanical processes, and are of very limited interest to the calculator, whereas multiplication is the key to all those properties which arouse his interest and stimulate him to establish the calculating habit. The differences between odd and even numbers, the properties of prime and composite numbers, as well as of squares, cubes, and other powers, series of all sorts, 2-figure endings, casting out the 9's, and the like, all grow directly out of multiplication.

The *methods* used in mental multiplication are various. The simplest is, as we have seen, a more or less abbreviated process of *counting*. It is of course difficult to say just which individual calculators remained permanently in this stage; we have seen, however, how large a part this counting process plays in the writer's own case, and it is not unlikely that this was the method of both Fuller and Buxton.

Several of the prodigies began multiplying at the *left*, instead of at the right as in the ordinary written method. The advantage of this procedure, as Colburn and Bidder explain, is that the larger figures first obtained are easily remembered, because ending with so many zeros, and that there is at any given stage of the work only one result to keep in mind. The result is changed, to be sure, by each new partial product incorporated with it; but the great difference between written

and mental arithmetic is that whereas in the former it is easiest to record the partial results and later combine them all at once, in the latter it is much easier to combine at each separate stage, and relieve the memory of the strain of remembering the partial results throughout the process. Probably in most cases the partial products actually *are* remembered, as by Inaudi, *e. g.*; but the fact that they *may* be forgotten if desired without interfering with the calculation, relieves the calculator of all anxiety in the matter.

Such is, in general, Bidder's explanation of this method. It is obvious, however, that no such carefully reasoned considerations can influence the calculator at the early stage when his methods are taking shape; and in mental arithmetic it is almost always easier to go on with an old method, however imperfect in theory, than to learn a new one. The real reason for Bidder's adoption of this method is doubtless the very simple one that it is more natural to begin with the first figure of the multiplier than with the last; if, however, the calculator should accidentally form the habit of beginning with the last figure, it is hard to see where any real inconvenience would result. In mental as in written arithmetic, much depends on custom and habit; it is hard to see any great difference in convenience between beginning at the right and beginning at the left, either in mental or in written multiplication. Of the two processes below,

$$
\begin{array}{r}
256 \\
243 \\
\hline
768 \\
1024\phantom{0} \\
512\phantom{00} \\
\hline
62208
\end{array}
\qquad\qquad
\begin{array}{l}
256 \\
243 \\
\hline
512 \\
1024 \\
\phantom{00}768 \\
\hline
62208
\end{array}
$$

it is custom, rather than the minute difference in convenience, that sanctions the first rather than the second.

There are two calculators, the younger Bidder and Diamandi, who are known to have used *cross-multiplication;* the elder Bidder, however, as we have seen, did not use this method, despite his son's statement to that effect. It is probably not a coincidence that both the users of cross-multiplication were of the visual type; for while this method has advantages which have already been explained, it is a radical departure from direct counting, and would naturally arise only as an abbreviation of *written* multiplication, or of mental multiplication done by the methods of written multiplication. In short, it is an "artificial" method of mental calculation. Cross-multiplication would be greatly facilitated by a mental picture

of the figures as arranged for written work; and we know that both these men possessed such mental pictures, though doubtless in a modified form. Diamandi learned mental arithmetic after his methods of written arithmetic had taken definite shape, and it is fairly safe to assume that the same was also true of the younger Bidder; this would imply, of course, that he was less precocious than his father.

Another method of multiplication possible for a visual calculator is that explained by Richard A. Proctor.[1] In brief, according to this explanation, each number is for the calculator a visual group or *pattern of dots*, discs, or the like, and multiplication consists in mentally juxtaposing or otherwise combining as many of these patterns of the multiplicand as there are units in the multiplier, and then rearranging the dots into a simpler pattern which indicates the product at a glance. Of course all sorts of short-cuts would come in with practice; but in principle the method is as indicated. The essential characteristic of these mental dot-patterns is their plasticity, the ease with which the pattern can be changed while the number of dots remains unaltered. Proctor tells us that as a child he himself acquired some little facility in mental arithmetic by this method, and probably with sufficient interest and practice it would give good results in the case of a visual calculator. Up to the present time, however, Proctor's case is the only one of the sort that has come to light, and there is no ground for extending this explanation to other known cases.

Another explanation of mental multiplication, proposed by Proctor and Scripture, attributes to the prodigies an *extended multiplication table*. Proctor conjectures that it may even reach (in the case of Bidder) to 1,000 x 1,000; Scripture, however, suggests only 100 x 100. On this theory, the multiplication of two 12-figure numbers would proceed by the division of each number into six 2-figure periods, or four 3-figure periods, which would then be used in the same way as ordinary mortals use single figures, giving the answer in 6 partial products of 6 operations each, or 4 partial products of 4 operations each, instead of 12 partial products of 12 operations each.

Doubtless such a method is theoretically possible, and would materially reduce the time required for multiplication; but can we attribute it to any calculator of whom we have specific knowledge? We know, of course, that any book-method or artificial aid of written or mental arithmetic can be utilized

[1] *Cornhill Mag.*, XXXII, 1875, p. 157; *Belgravia*, XXXVIII, 1879, p. 450.

with sufficient practice. Teachers have taught whole classes of pupils to multiply with the aid of more or less extended tables; Gauss used logarithms in his mental calculations; and the younger Bidder, Ferrol, and probably Huber's blind Swiss used mnemonics of some sort. The question is, however: Did such men as Buxton, Colburn, Mondeux, Dase, and Bidder—men who undoubtedly belong to the "natural family of great calculators," *i. e.*, whose methods of calculation took shape in the first instance independently of books—use multiplication tables reaching to 100 x 100, or even 50 x 50, or can their feats be explained without presupposing a table reaching beyond 10 x 10 or 12 x 12?

There are two ways in which such an extended multiplication table might be acquired. In the first place, the calculator might sit down and figure out the various products, either mentally or on paper, and then commit them to memory, a few at a time, until all were fully mastered. In this case the multiplication of 48 by 64 would take the mental form, "Forty-eight times sixty-four are 3,072," or, in a shortened form, the numbers 48 and 64, connected by the multiplication sign or the words "multiplied by," would call up directly the idea of 3,072, without any intermediate calculation, either clearly conscious or automatic; just as 9 x 7 means 63 for the ordinary man, without the intervention of any of the other multiples of 9 or 7. Fatigue or lack of practice might render the process slower, but would not introduce any intermediate links of calculation. In the second place, the calculator might, with practice, be able to multiply 48 x 64 so readily and rapidly, by more or less automatic processes, that he would get the answer, 3,072, as quickly as if he had relied on a direct act of memory; and if the process of calculation happened to be mainly or wholly automatic, he might even be ignorant of its existence, and suppose he had actually found the result by direct and unaided memory. At times, however, brief flashes of these intermediate calculations would pass through his mind; and when he was tired or out of practice, his calculations would not only be slower, but would be of a more clearly conscious character. The intermediate links, which had been made automatic and half unconscious by practice, would return as soon as fatigue or disuse reduced the calculator's speed.

In this second case, however, it is evident that the whole 5,000 or 10,000 entries of a multiplication table to 100 x 100 cannot properly be said to exist already computed in the calculator's mind. We shall have to discuss a little later the various ways of shortening the calculation on the psychological side; but in dealing with the enlarged multiplication table theory, we must insist that the only legitimate interpretation of the

theory is that such a table is *deliberately committed to memory* by the calculator, and not reached in particular cases by an abbreviated process of calculation whose omitted links could return under any ordinary circumstances. Fatigue or lack of practice might prolong the time of unmeditated recall, but could never interpolate into it even the briefest flashes of calculation. If, then, no specific evidence is at hand that such an enlarged table actually *was* used by any of the "natural calculators,"—and no such evidence has been adduced by the advocates of the theory,—we cannot accept this explanation unless it is shown, either that the enlarged multiplication table is a more natural method than the smaller table, or that the actual achievements of some or all of the calculators cannot be explained in any other way. Actually, the latter of these arguments is the one on which the defenders of the theory seem to rely; it will be safest, however, for us to examine the former as well.

In the first place, then, it can hardly be claimed that a multiplication table extending much beyond 10 x 10 is either natural or useful in the early stages of mental calculation, if the calculation arises in the natural and not in the artificial way. The child who becomes a calculator begins to multiply soon after he learns to count, certainly before he has learned to count beyond 1000, and hence before his multipliers have exceeded 31. It will hardly be argued that up to this point a table beyond 10 x 10 would be of any use. Hence *the calculator's habits and methods of multiplication are definitely formed before a table beyond 10 x 10 is needed.* Such a table serves all the purposes even of written multiplication for most of us; in fact, even if we know the multiplication table to 12 x 12, we rarely use it beyond 10 x 10. How many of us in multiplying 412,976 by 3,128, for instance, would think to treat the 12 in either the multiplicand or the multiplier as a single factor? It is only where the multiplier itself is simply 11 or 12 that the 12 x 12 table excels the 10 x 10 table for any practical purpose. Hence, unless an enlarged table extended all the way to 100 x 100, its utility would be relatively slight. In mental multiplication, too, we know that Inaudi, who could multiply 6 figures by 6 figures, used a table reaching only to 10 x 10. We are thus brought to our second question: In multiplication by 12-figure numbers or larger, need we presuppose a table extending beyond 10 x 10 in order to explain the actual achievements of known calculators? Must the calculator enlarge his table as the size of the numbers he uses is increased, or does he simply depend on practice, and on new short-cuts other than enlargement of his multiplication table, to extend his powers and increase his speed?

Bidder's answer to this question in his own case is explicit.

He says:[1] "Now, for instance, suppose that I had to multiply 89 by 73, I should instantly say 6,497; if I read the figures written out before me I could not express a result more correctly, or more rapidly; this facility has, however, tended to deceive me, for I fancied that I possessed a multiplication table up to 100 x 100, and, when in full practice, even beyond that; but I was in error; the fact is that I go through the entire operation of the computation in that short interval of time which it takes me to announce the result to you. I multiply 80 by 70, 80 by 3; 9 by 70, and 9 by 3; which will be the whole of the process as expressed algebraically, and then I add them up in what appears to be merely an instant of time."

This testimony is unequivocal; Bidder, as Scripture admits, certainly did not have a multiplication table extending beyond 10 x 10, yet he was able, on one occasion at any rate, to perform a 12-figure multiplication. Furthermore, Bidder's calculations were so rapid and automatic that he was himself deceived in this matter, and thought he actually had such a multiplication table as Proctor attributes to him. Even if we grant that when Bidder wrote this account, in his 50th year, his powers had slightly diminished, with the result that his calculations were somewhat less automatic than in his youth, it is clear, nevertheless, that even in his prime as a calculator he depended on calculation, not on a memorized multiplication table beyond 10 x 10.

What shall we say, now, of other calculators? Fuller, the man who required 2 minutes for a simple calendar problem, may be dismissed at once; he needed no enlarged multiplication table. Buxton, Scripture tells us,[2] "preserved the several processes of multiplying the multiplicand by each figure of the lower line in their relative order, and place as on paper until the final product was found." The reference for this passage is not given. The statement, however, is directly contradicted by Buxton's own account of his method of multiplication, according to which, as we have seen, he multiplied by 5, then by 20, then by 3 to multiply by 300.[3] It is safe to omit Buxton, therefore, from the list of users of large multiplication tables. Colburn's account of his method of multiplication[4] is almost identical with Bidder's, and leaves no doubt that he multiplied

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[1] *Proc. Inst. C. E.*, XV, p. 256.

[2] *Op. cit.*, p. 58. On this statement, which is almost certainly incorrect, Scripture bases his belief that Buxton had a good "imagination," *i. e.*, belonged to the visual memory type.

[3] In the light of this example, which he himself quotes (p. 48), it is hard to understand *why* Scripture thinks (p. 46) we can presuppose the enlarged multiplication table in the case of Buxton.

[4] *Memoir*, pp. 189-191.

by but one figure at a time. Mondeux, we are told,[1] "actually possessed part of such a table." Here again, unfortunately, the reference is omitted. The only statement in Cauchy's report that looks like a foundation for this assertion is that Mondeux "knows almost by heart the squares of all whole numbers under 100." But, as we have already seen, the powers of numbers strongly attract the interest of the calculator at an early stage; hence in the absence of other evidence it is not legitimate to infer the presence of a complete multiplication table, or any essential part of it, from the presence of a table of squares. Inaudi's methods are fully described by Binet, and show that he used a table extending only to 10 x 10. Diamandi and the younger Bidder used cross-multiplication, and there is no definite evidence that they used more than one figure at a time. Safford so closely resembles Bidder and Colburn that, in the absence of evidence to the contrary, we may safely assume that his method of multiplication was the same as theirs. Zaneboni did not possess a systematic table extending to 100 x 100, though he knew many squares and a few scattered products beyond 10 x 10.[2] Concerning Ampère so little is definitely known that it is idle to speculate about his methods; but this is hardly a proof that he used an enlarged multiplication table.

There remain, then, only Gauss and Dase. It must be admitted that these men *may* have used enlarged multiplication tables; the assertion cannot be disproved on the basis of available evidence, and both men seem to have excelled practically all other calculators. But while Gauss undoubtedly began as a "natural" calculator, he afterwards also used logarithms and other "artificial" methods in his mental calculations; so that, even if we grant—though there is no known evidence for the view—that he used a large memorized multiplication table, his case affords no basis for inferences concerning the procedure of other calculators less gifted mathematically. Thus the case of Dase alone among the "major calculators" remains for further consideration.

We have already seen that in rapidity and extent of calculation Dase stands in a class apart. It would therefore be unsafe to say dogmatically that his methods did not differ from those of other calculators. Even in his case, however, the ordinary methods are sufficient to explain all his feats, without recourse to the extended multiplication table theory. Inaudi could perform a 2-figure multiplication, by the ordinary method, in 2 seconds, the partial products involving 6 figures, or at the rate

[1] Scripture, *op. cit.*, p. 46.
[2] *Riv. sper. di Fren.*, XXIII, 1897, p. 411.

of 180 figures of partial product per minute. Dase required 54 seconds for an 8-figure multiplication, with not more than 72 figures of partial product, or 80 figures per minute, a falling off of about 55% from Inaudi's speed in the shorter operation. For any other calculator a much greater falling off would naturally be expected; but in a 40-figure multiplication, performed in 40 minutes, Dase was able to maintain a speed of 41 figures of partial product per minute, and in a 100-figure multiplication a speed of about 20 figures per minute, in complete defiance of the rules that hold for other calculators.[1] Hence even if we grant that Dase divided the numbers into 3-figure periods, he may well have multiplied within those periods by the ordinary method, without having recourse to a table larger than 10 x 10.

There is no warrant, then, for supposing that any of the prodigies except Gauss and Dase used a multiplication table larger than that of ordinary mortals; and even in these two cases there is no direct evidence, only a bare possibility; all their known feats are explicable on the supposition that they used the small tables of Bidder, Inaudi, and the rest. We may therefore dismiss the theory of enlarged multiplication tables,

[1] In Binet's tests Inaudi performed a 2-figure multiplication in 2 seconds, 3-figure in 6.4 seconds, 4-figure (the limit of his ordinary stage exhibitions) in 21 seconds, 5-figure in 40 seconds, but 6-figure in 4 minutes, or 240 seconds. This sudden increase in the time, 6 times as long for 6 figures as for 5 figures, seems to indicate that in passing from 5 to 6 figures Inaudi exceeded the limits of his ordinary practice, and became confused. Bidder (*Proc. Inst. C. E.*, XV, p. 256) suggests that in cases where the multiplicand contains the same number of figures as the multiplier, the difficulty should increase roughly as the fourth power of the number of figures in the multiplier, since the number of partial product figures involved increases as the square of the number of figures in the multiplier, while the difficulty of remembering the larger numbers also increases roughly as the square of the number of figures in the multiplier. On this basis, the time for 6 figures ought to be about twice as long as for 5 figures, whereas with Inaudi it was 6 times as long, or 3 times the theoretical speed. In Dase's case, however, the results are all in the other direction. In passing from 8 figures to 40 figures his time increased only about 44-fold, or 1-14 the theoretical increase (625 times); and in passing from 40 figures to 100 figures the increase in time was only 13 instead of 39 times, or ⅓ what we should expect on Bidder's theory. Of course these comparisons cannot pretend to be exact; still they are interesting, as showing how little Dase's powers were strained by increase in the size of the numbers. Nowhere in his calculations is there any indication of such confusion as overcame Inaudi in passing from 5 figures to 6 figures. In fact, we may well suppose that only physical fatigue could limit the extent of Dase's calculations in a single sitting. If he could, like Buxton, after a night's rest resume his work where he left off on the previous day, it is safe to assume that even a 200-figure multiplication would hardly have been beyond his powers.

at least until its advocates have brought forward further and more definite evidence than any that has yet been produced.

*Factoring* was a favorite problem with Colburn, Bidder, and Safford; none of the others, however, are known to have explored this field. Colburn and Bidder, and doubtless Safford [1] as well, used 2-figure endings in solving problems of this class.

We have seen that *square and cube root* problems were favorites with several of the prodigies. Here, as in multiplication, Dase towers above all the rest; he could extract the 30-figure square root of a 60-figure number in an "incredibly short time," and the 50-figure square root of a 100-figure number in 52 minutes. His method was probably similar to the ordinary written method, but with the short-cuts suggested by his great familiarity with large numbers. Whether he found part of the numbers by simple division, or preferred to continue his approximations in the ordinary way to the end of the process, is not known. He was almost certainly acquainted with some of the properties of 2-figure endings,[2] and may have used them in finding the roots of small numbers. More probably,

[1] The evidence in Safford's case is as follows: (1) The general similarity between Safford on the one hand, and Colburn and Bidder on the other. All three specialized in evolution and factoring, where 2-figure endings are peculiarly helpful; Colburn and Bidder tell us explicitly that they used these endings; hence it is probable that Safford did also. (2) Safford extracted the cube roots of three 7-figure numbers "instantly." Now the ending method is instantaneous for numbers up to 9 figures, and is the only known method that is instantaneous. (3) When asked the cube root of 3,723,875, he answered, "155, is it not?" whereas in the case of numbers not ending with 5 his answers were categorical. Now for most odd endings there is only one possible cube root; but for the ending 75 there is a choice between 15, 35, 55, 75, and 95. A slight hesitation between 135 and 155, or 155 and 175, would therefore be natural if he depended on the ending primarily, whereas if he depended on memorization of other cubes, or direct calculation, multiples of 5 would be no harder than other numbers.

[2] The evidence in this case is that Dase was fond of finding 5th powers, because the last figure was always the same in the 5th power as in the given number. (Gauss-Schumacher *Briefwechsel*, V, p. 382.) Now the 5th power also has the property that, in many cases, the last figure alone of the given number determines the last *two* figures of the 5th power. Any one who had found 5th powers often enough to acquire a preference for them could hardly fail to note this property, which in turn might easily lead to the discovery of the properties of 10th and 20th powers, especially since 10th and 20th powers are also themselves 5th powers. But since at the 20th power there are only four possible endings,—00 for numbers ending with 0, 25 for numbers ending with 5, 01 for all other odd numbers, and 76 for all other even numbers,—and since after the 20th power the cycle of endings repeats,—the 21st power ending being usually the same as the 1st, the 22nd always the same as the 2nd, etc.,—the properties of the whole cycle are readily investigated, and would be almost certain to attract the calculator's attention, as soon as he had become really interested in 5th powers.

however, he knew by heart the squares of all numbers at least up to 100, in which case 2-figure endings would be of less use to him. In dealing with numbers of 60 or 100 figures they would be practically useless, for the reason that where the given number is as long as this, it is usually selected by the questioner at random, rather than formed by laboriously squaring a large number. Hence the given number will rarely be a perfect square, and the method of endings will not give even the last two or three figures. Whether it can be used at any of the intermediate stages of the work is doubtful.[1]

Most of the other prodigies who have made a specialty of square and cube root problems either depended on 2-figure endings, resorting to guess and trial when the given numbers were not perfect squares or cubes, or else kept in mind the squares and cubes of many or most of the numbers up to 100 or beyond. Such methods would work fairly well when the root contained not more than 5 or 6 figures. Answers can be obtained "instantly," however, only by the method of endings, and even by that method only when there are not more than 3 figures in the root; though the method might be made "instantaneous" for 4-figure roots of exact squares and cubes if the calculator had committed to memory the squares and cubes of all numbers under 100.

Inasmuch as the part played by 2-figure endings has hitherto been little recognized by students of the mathematical prodigies, it seems worth while here to determine, if possible, just how widely they were used by the different calculators. Colburn, Bidder, Safford, Zaneboni, and the present writer, we have seen, used them more or less freely. There is some evidence, not conclusive, however, that Dase was familiar with their properties. The daughter of the Countess of Mansfield extracted readily the square and cube roots of 9-figure numbers, and may have used this method. Gauss was familiar with many propositions in number-theory, of which the properties of 2-figure endings are only special cases; it would hardly be safe to infer, however, that he used these properties extensively in his mental calculations. A bare possibility exists in the case of one or two other calculators—Mondeux, for instance—that

[1] John Wallis (1616-1703), according to letters of his reprinted in the *Classical Journal*, XI, 1815, p. 179, and in the *Spectator*, LII, 1879, p. 11, extracted the square root of 3,00000,00000,00000,00000,00000,00000, 00000,00000 mentally, and on another occasion the square root of a 53-figure number. The third figure of the first answer is wrongly given in both journals as 7 instead of 3; as the next 3 or 4 figures are correct, however, for $\sqrt{3}$, the error is probably only typographical. In the *Spectator* the first problem is wrongly given as 30000, etc., instead of 3,00000, etc. Scripture (*op. cit.*, p. 38) carries over both errors from the *Spectator* without comment.

the properties of the endings were more or less fully known. Without seeking to exaggerate the importance of these endings, therefore, we may safely say that of the calculators we have studied, about one in every four was familiar with them. Their importance for us here, however, is rather as an illustration of the sort of numerical symmetries and properties which arouse the interest of the calculator than as new and valuable discoveries; in fact, from the mathematical standpoint they are trivial and of very limited interest. To answer questions in evolution and factoring, the mathematician would turn to his tables of factors or roots, or to a logarithm table; he would regard the properties of the mathematical prodigy's 2-figure endings as unimportant special cases of more general propositions in the theory of numbers. Up to the present time, then, these endings are of merely curious interest except in connection with mental calculation; though it is conceivable, of course, that if a new and comprehensive theory of their properties were worked out, it might find a subordinate place in the theory of numbers.[1]

It will be noted that the term "ending" has been used in the present paper as a synonym for "2-figure ending," the implication being that no other kinds of endings are of importance in mental calculation. It is easy to see why this should be so. The properties of 1-figure endings, or last figures, are so simple that many of them are familiar to the average schoolboy, and they tell so little about the answer to a given problem that their utility in mental arithmetic is negligible. To study 3-figure endings, on the other hand, would involve a large number of 3-figure multiplications; so that here, as in the case of the multiplication table, if the endings are used at all, the use of the smaller (2-figure) endings becomes a fixed habit long before larger ones could be of any real use. The only known exception occurs where the given power ends with 5; in this case there is an unusually large number of possible roots, and in seeking for some way of choosing between them, some of the calculators (Bidder, for example) noted what 2-figure roots corresponded to the various 3-figure endings for multiples of 5. It is obvious, however, that the use of 3-figure endings in this exceptional case would not warrant the inference that they were used in any other case; and Bidder's testimony shows clearly that he, at any rate, made no further use of them.

[1]In one sense, however, these endings may be regarded as the properties or functions of numbers analogous to logarithms, and previously unknown to mathematicians, which some writers suggested might explain Colburn's rapidity in calculation, especially in evolution and factoring. (*Phil. Mag.*, XL, 1812, p. 125; *Analectic Mag.*, I, 1813, p. 128.)

Many simple *algebraic problems* were solved mentally by the different calculators. Those who had a general mathematical training of some sort—Ferrol, Safford, and Gauss, for example—doubtless solved these problems by algebraic methods. Many of the others, however, depended on simple trial, or on little tricks which are readily discovered for certain classes of problems, and need not be dwelt on here. It is safest to assume that the method was one of simple trial in all cases where there is not definite evidence to the contrary.

Bidder's method of solving *compound interest* problems has already been described. In this particular branch he seems never to have had a rival.

*Arithmetical Association.* Under this caption Scripture,[1] in part following De Morgan and others, has given an account of some of the ways in which the psychological processes of mental calculation can be shortened. He points out, in particular, that with practice the mental calculator can easily train himself to omit useless words, and think only of the *numbers* concerned; thus instead of saying, "3 and 4 are 7," "3 times 7 are 21, put down 1 and carry 2," and the like, he can learn to say simply, "3, 4, 7," "3, 7, 21, 1, 2," etc. As De Morgan expresses it,[2] "Don't *say* 'carry 3,' but do it." It is evident, furthermore, that as the process gradually becomes more and more familiar and automatic, many of the intermediate steps of the computation may partly sink into the background of consciousness, perhaps even disappearing altogether from the field of attention; thus, as we have already seen, the intermediate links in Bidder's 2-figure multiplications were at one time so completely automatic that he believed they were altogether absent, and supposed he possessed a multiplication table extending to 100 x 100 or even beyond. Moreover, where the given numbers are familiar, only part of a number may be clearly conscious, even when the whole number functions in the calculation. Thus in the present writer's case the ending of a number may for many purposes completely replace the number itself. A further abbreviation which may be noted is that, in the case of an auditory calculator, any chance visual associations which may be present in the early stages of his calculations, but which play no active part in them, may gradually drop out as his skill increases. The same thing may happen, in certain cases, with the associated motor tendencies, if they are not too strong. These psychological short-cuts, in

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[1] *Op. cit.*, p. 42 ff.

[2] *Elements of Arithmetic.* London, 1856, p. 164. In this work (pp. 161-5) will be found a very clear account of the natural counting process, with some of its modifications and short-cuts, by which the "natural" mental calculator is developed.

connection with the mathematical short-cuts to which we have already referred, explain the great rapidity attained by some of the prodigies in their mental operations.

A certain amount of caution is necessary, however, in attributing these various short-cuts to individual calculators, except where we have their own explicit testimony as a check. Binet was able to establish that Inaudi, in his mental multiplications, made little, if any, use of these short-cuts; and simple repetition of the unabbreviated calculations will produce a considerable increase of speed without further aid in the way of mathematical or even psychological short-cuts of the sort just described. We have seen how little ground there is for attributing extensive multiplication tables to most of the prodigies; yet some writers have supposed that this was the simplest, if not the only, explanation of the speed with which many of the prodigies could calculate. In theory, there is no definite limit to the short-cuts possible in mental calculation, and it is evident that in the case of such a man as Dase a good many of these short-cuts must be actually realized in practice; at the same time, it is not safe to attribute any *particular* short-cut to a given calculator unless specific evidence is at hand that he actually used that particular method and not some other equally rapid.

*Memory.* (*a*) *Memory versus Calculation.* It has become customary, in the literature on mathematical prodigies, to distinguish more or less sharply the parts played by memory and by calculation proper in the various operations, especially where the numbers dealt with are very long. Bidder tells us that only the limits of his memory would stand in the way of performing immense calculations in an incredibly short time. Buxton was excessively slow in calculating, but had such a tenacious memory that he could work on a problem for weeks, and so solve almost any problem, long or short, that happened to arouse his interest. Dase, according to this view, differed from ordinary mortals mainly in the possession of a wonderful memory; but as far as the calculation proper is concerned, we have the authority of Gauss for the statement that Dase's 100-figure mental multiplication, which required 8¾ hours, could be done on paper in perhaps half the time.[1] This view implies, then, that the processes of the calculator in a very long mental multiplication are essentially the same as in a very short one, being in either case substantially those of written multiplication; in other words that the work, computed bit by bit in the long as in the short multiplication, is retained in the memory in a more or less isolated and mechanical fashion, just as

[1] Gauss-Schumacher *Briefwechsel*, V, p. 297.

on paper the partial products are first computed and recorded, figure by figure, and then added. Practically, then, the view is a sort of common-sense faculty psychology, seeking to justify itself by pointing to the difference in speed between mental and written multiplication.

Now it can hardly be questioned that the mathematical prodigy's figure-memory is superior to the ordinary man's. Dealing constantly with figures, the mental calculator learns to assimilate them readily. A 20-figure number, which for most of us is a meaningless string of figures devoid of interest, for him "makes sense," and so is easy to learn, just as a page of French is more easily learned by a Frenchman than by a foreigner who knows little or nothing of the language. Hence we may safely assume that the calculator's figure-memory will outstrip his powers of calculation; that is to say, by the time he can mentally multiply two 6-figure numbers, let us say, he will have little or no trouble in remembering almost at a glance, or after a single hearing, two numbers of 7 or 8 figures each. This is on the supposition, of course, that he has never practiced remembering long numbers merely for their own sake; we must here distinguish those calculators, like Zaneboni, who were fond of committing statistics or very long numbers to memory without any immediate intention of using these numbers in mental calculations, from those who made no effort to develop their memory for figures except in the service of calculation.

The question is, then: Will special practice in memorizing numbers, apart from their use in calculation, extend the powers of the prodigy in calculation proper? In other words, does he use the same mathematical and psychological processes in a long multiplication as in a short one, except that in the former he stops every now and then to deposit some of his partial results temporarily in his memory, later returning and picking them up in order to unite them into a final result? In written arithmetic we compute a series of partial results, register these results on paper at each stage, and later come back to unite them, by a separate operation, into the total product; is this the general scheme of mental multiplication also?

It would perhaps be unsafe to answer this question dogmatically in the negative; at the same time, we should have to distinguish as an "artificial" calculator any one who proceeded in this way, even granting that it has ever been done, which is not known to have been the case with any recorded prodigy. We have already seen that both the mathematical and the psychological processes involved in mental multiplication differ considerably from those of written multiplication. Moreover, the two have quite distinct origins; mental arithmetic grows

naturally and independently out of counting, written arithmetic out of more or less arbitrary rules learned from teachers and books. Any argument based on the methods of written multiplication, therefore, or on the difference in speed between the two dissimilar operations of mental and written arithmetic, must be accepted with considerable caution. The fact that on paper the computing and the recording are two distinct operations does not prove that in mental arithmetic memory and calculation are two separate faculties, and that when the numbers to be multiplied are very long, the difficulty is all on the side of memory and not at all on the side of calculation.

Even in Buxton's case, where the discrepancy between memory and calculating power seems as clear as anywhere, we cannot give the credit to memory, as opposed to calculation. Buxton's calculations, to be sure, whether long or short, were excessively slow; but so was his memory (in the sense of power of acquisition of figures): we are told that he comprehended even (simple) arithmetical questions "not without difficulty and time." [1] Like Bidder and many—perhaps most—other calculators, he began work at the left of the numbers, adding up the partial products as he went along, and not waiting until all were obtained. Moreover, we have seen fairly good reasons for supposing that he worked by a process of modified counting, rather than by multiplication proper, dividing the work into stages in his own peculiar way, but for the most part keeping in mind only one or two partial results, and using them in calculation as soon as possible. This means, however, that his processes in a long multiplication were not simply the same as in 2-figure multiplications, plus a tenacious memory; he really *calculated* with the *large* numbers, and in dealing with them doubtless used some special short-cuts, though far fewer than most other calculators. In short, he really was, in his way, a "great calculator," not a "little calculator" with a "big memory." He calculated with large numbers because they had an interest and a meaning for him which they have not for the ordinary man. The difference between the man who can unravel a half-page sentence of technical German and the one who can scarcely understand a two-line sentence is not primarily a matter of memory, but of interest and meaning; and the same holds good of the calculator who handles large numbers, as compared with the one who does not. Zaneboni had an unusual interest in memorizing figures for their own sake, and Inaudi had a highly developed figure-memory, yet we do not read that these men made a specialty of long multiplications;

[1] *Gent. Mag.*, XXIV, 1754, p. 251.

their figure-memory as such and their figure-memory as subservient to their interest in calculation were quite distinct.[1]

We cannot agree with Binet,[2] therefore, that mental calculation combines the two distinct and independent elements of memory and calculation, of which memory is really distinctive in the case of the great calculators, the one characteristic in virtue of which they are inimitable and indefinitely superior to the rest of mankind. Inaudi, to be sure, was inferior to the department-store cashiers on short multiplications, because he had not practiced them as much as his rivals had; but his superiority in long multiplications is equally due to practice in calculation, not to superior memory as such. Some of his calculations were slower than those of a good computer on paper, others were faster; but the mental and even the mathematical processes are so different in the two cases that no value attaches to the comparison. His skill in memorizing long numbers was the result of practice, perhaps in the service of addition, perhaps for its own sake, but was not the secret of his calculation. That he remembered the 200 or 300 numbers used in one of his public exhibitions was due to his *interest* in the figures, on account of their connection with his calculations; where that interest was lacking, he could remember only a third as many figures in the same period of time. And how little connection his figure-memory had with his calculations proper is shown by the fact that when he exceeded the range of his practice in *calculation*, and passed from a 5-figure to a 6-figure multiplication, he required six times as long,—a result inexplicable, in a man who could retain 42 figures on a single hearing, if memory is the real secret, but much less surprising if practice in calculation is the important thing. In short, figure-memory is important in the psychology of mental calculation only in so far as it stands *in the service of calculation* and intimately bound up with it; to make a sharp distinction between the two, and lay the emphasis on memory as *opposed* to calculation, is to be led astray by a distinction of common-sense psychology whose usefulness has long since been outgrown.

(*b*) *Memory type.* We have seen that up to 1892, when Inaudi was shown to belong to the auditory type, it was generally taken for granted that all mathematical prodigies were visual. Of the two important calculators since discovered,

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[1] In the case of long additions, such as Inaudi's additions of 21-figure numbers, the distinction between memory and calculation may at first seem clearer than in multiplication. Actually, however, the two are, if anything, more closely connected in addition than in multiplication; for the calculation involved is so simple and automatic that it probably always begins during the process of memorizing the second number.

[2] *Op. cit.*, p. 194.

Diamandi is undoubtedly visual, and Zaneboni is so described by the authors of the article in the *Rivista di Freniatria*, though, as we shall see, his case is somewhat doubtful. Up to the present time, then, Inaudi has been supposed to be the only non-visual calculator on record; no one seems to have raised the question whether all the supposedly visual calculators actually were visual and not auditory. It is proposed here to make an attempt to answer this question; and while it will, of course, not be possible in all cases to reach perfectly definite conclusions, we can at least make a start in the right direction, and put the whole subject on a more satisfactory and critical basis.

First, however, it will be necessary to define our terms. It should be understood at the outset that no absolute line can be drawn between the two types. In ordinary life the most common type would probably be the "mixed" type. But it will be impossible, on the basis of the available data, to enter upon this question of the *general* memory type of the different calculators. The most we can hope to do is to determine to which type they belonged *in their mental calculations*; and while it is probable that this will usually coincide with their general type, the fact can by no means be taken for granted. Fortunately, however, it is much easier to decide in a narrow field than to decide the general question; and in calculation, especially, there are a number of fairly definite and reliable indications which will render the task somewhat easier than it would be in many other fields.

By a *visual* calculator, then, we shall mean one in whose mental calculations visual images of some sort play an essential part. Auditory elements will probably always be present besides, in greater or less degree; but if the visual elements play an essential rôle, we may call the calculator visual, since any more minute classification would be impracticable without more detailed information than we possess concerning the individual men. The visual elements may take various forms. The calculator may see numbers as dot-patterns, as Proctor did. He may have a number-form. Or he may see the figures of his calculations written or printed before him, more or less distinctly, but not necessarily in complete detail, since short-cuts and abbreviations may be connected with visual images as well as with any others. In other words, the visual image need not be a photographic reproduction of the complete written calculation; if it is essential, in however schematic and abbreviated a form, its presence is the mark of a visual calculator.

Inasmuch as written figures are the commonest form of the visual image,—Proctor is the only calculator known to have used dots in his actual calculations,—it is to be expected that in most cases the visual calculator's methods of calculation will

not take shape until after he has learned to write figures. Since the visual image is simultaneous, while the auditory image is successive, it will be easier for a visual than for an auditory calculator to reverse his image, and call off a number backwards. Hence the ability to read off a number from memory *either* forwards or backwards will establish a presumption in favor of visual type. This is not a conclusive indication, however, since with practice an auditory calculator also could learn to reverse his numbers, especially where they were short. At the same time, the reversal is so much easier for a visual than for an auditory image that it must be considered as establishing at least a presumption in favor of the visual type, especially when corroborated by other indications.[1]

We may call a calculator *auditory* (or auditory-motor), on the other hand, when there is some definite reason for believing that visual images do *not* play an essential part in his mental calculation, or that motor tendencies are closely associated with it. Visual images need not be completely absent, especially in the early years of a precocious calculator; but if they were present at the start, we must have some reason for believing that they became unimportant as his proficiency increased. Similarly, the motor tendencies may, with practice, partly or wholly drop out; but in this case they will tend to return when new and unpracticed facts are attempted. In consequence of the *verbal* nature of the *counting* process in which natural mental arithmetic begins, it is hard to conceive of a visual calculator in whose operations words (and hence auditory images) do not play a part of greater or less importance; it is much easier, on the other hand, to conceive of an auditory calculator in whom visual elements are almost wholly absent, except in the form of an occasional chance by-play of associations which have no essential function in the calculation proper. If this is the case, however, it constitutes a presumption that any given calculator is auditory unless evidence exists to the contrary, and throws the burden of proof upon the visual side.

The most definite indications of auditory type are as follows: (1) A tendency to articulate during calculation, especially in relatively unfamiliar operations; this may be regarded as practically conclusive evidence of auditory (or auditory-motor)

[1] Curiously enough, however, we have no specific record of the possession of such a power of reversing numbers by either Diamandi or the younger Bidder, the only two calculators for whom we have perfectly definite and unequivocal evidence of visual type; though it is practically certain that Diamandi, at any rate, possessed the power in some degree, since after learning a square of figures by rows from left to right he could, with little difficulty, repeat them in ascending or descending columns.

type. (2) The absence of any indication of visual elements in the prodigy's mental calculations. This argument, however, is of no practical weight except in cases where we have a fairly full account, from the prodigy himself, of his methods of calculation. Where this is the case, it is fairly safe to assume that at least some of the various indications of visual type will be present in the narrative if the calculator is really visual. Colburn and Bidder are the only men who have left us such narratives, and in neither case do we find any reference to visual processes; hence a strong presumption exists that both these men were auditory. (3) General resemblance to known auditory calculators. This is not a very satisfactory indication, in general, unless the resemblance is close in several important respects; but in certain cases, owing to the meagreness of the sources of information, it is almost the only indication we have. It is more reliable, of course, if it covers several indications which, though perhaps each inconclusive by itself, may have a cumulative weight.

Let us now examine the evidence in the case of the different individual calculators, in each group taking those cases first where there is least room for difference of opinion.

As *visual* calculators we may name at once the younger Bidder and Diamandi, both of whom possessed number-forms and used cross-multiplication; the younger Bidder, moreover, made the not very profound observation that he could "conceive no other (non-visual) way possible of doing mental arithmetic." Diamandi took up mental calculation after leaving school, at the age of 16, and hence after learning to read and write; the same is probably true of the younger Bidder, though inasmuch as his number-form shows the influence of the clock in the circular arrangement of the numbers up to 12, he may possibly have learned numbers visually from this source before he learned the alphabet.

Whether *Dase* was of the visual type it is difficult to say. He may have learned to read and write before he began to calculate,[1] though this is not certain; he could repeat numbers backwards with great ease; he could learn a number of 12 figures almost at a glance; and his skill in rapid visual counting was remarkable. All these indications point in the visual direction, yet they are by no means as satisfactory as the proofs in the other two cases. We have already seen that Dase stands in a class by himself, and that an enormous amount of practice is presupposed by his feats, whatever his memory type; it is

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[1]He attended school at the age of 2½, but attributed his skill in calculation to later practice and industry rather than to his early schooling.

not impossible, therefore, that he was auditory, since with sufficient practice an auditory calculator could learn to reverse numbers readily and to count objects at a glance. Robert Houdin and his father, we know, acquired by practice a considerable power of rapid visual counting. Moreover, it does not follow that a man who by practice has developed a power of visualization in this field will likewise use visual images in mental arithmetic, especially since the precise amount of visualizing power needed for Houdin's feats is not at all easy to determine. But in order to avoid one-sidedness we may give the visual theory the benefit of the doubt, and call Dase a visual calculator, bearing in mind, however, that the evidence is by no means as satisfactory in his case as in the case of Diamandi or the younger Bidder.

The case of *Zaneboni* is likewise somewhat doubtful. He began to calculate at the age of 12, after learning to read and write, and used written lists in some of his memory feats; he also seems to have made more or less use of visual images in answering questions concerning railway distances and the like. He could repeat a memorized 256-figure number almost as readily backwards as forwards. It is not at all clear, however, to just what extent he used visual images in his mental calculation proper, as distinguished from figure-memory proper; and in the experiments of Guicciardi and Ferrari there are some indications which clearly show a leaning in the auditory-motor direction. If we were discussing his memory-type in general, instead of his type simply in calculation, we should undoubtedly be safe in describing him as "mixed"; but the definitions we have adopted of the terms "visual calculator" and "auditory calculator," do not admit any intermediate type, since we have agreed to call a calculator visual if visual images play any essential part in his calculations. A detailed examination of the available data concerning his type as a calculator would take up far more space than its importance would warrant, and even then would leave us in doubt just where to place him. It seems wisest, therefore, to give the visual theory once more the benefit of the doubt, and call Zaneboni a visual calculator, despite the inconclusiveness of the existing evidence. We find so far, then, four visual prodigies,—the younger Bidder, Diamandi, Dase, and Zaneboni,—of whom only the first two, however, are above suspicion.[1]

As *auditory* calculators we may name at once Inaudi, Ferrol, who in ordinary life was an abnormally poor visualizer, Saf-

[1] The name of Proctor has been omitted from this list, owing to the absence of information concerning the extent of his calculating power; there is no doubt, however, that his calculations, as far as they went, were visual in the sense already defined.

ford, who talked to himself when originating new rules, and the present writer. The cases of certain other calculators who were almost certainly auditory require some discussion, since at first sight the evidence appears to be conflicting.

That *Bidder* was of auditory type is shown by the following considerations. (1) He learned to count and calculate before learning to read and write, and constantly emphasizes the importance of this fact in his lecture before the Institution of Civil Engineers. After learning to count, he taught himself the multiplication table to 10 x 10 by means of rectangles of shot; but he expressly speaks of *counting*[1] the shot after thus arranging them, not of seeing a picture of them, or of grouping them in visual patterns. Again, when he recommends this method of teaching mental arithmetic,[2] he lays stress on arranging the actual objects and then counting them, not on seeing such vivid mental pictures as would render the presence of the actual objects superfluous. Hence we are warranted in assuming that if Bidder had visual images of his shot and marbles at first, they played no part in his actual mechanism of mental calculation, and later dropped out. If this is the case, then the use of pebbles and the like in childhood is not, as it would at first seem, a proof of visual type.

(2) Bidder's reiterated emphasis on the *teachableness* of mental arithmetic, by his methods, to any person of average ability,[3] is itself a fairly good indication of his auditory type. Any one can learn to count in auditory terms, but not every one can learn to visualize numbers, to "have a good imagination" for them. If visual aids played any essential part in his mental calculations, then, we should expect him to stipulate that the mental arithmetician must possess a certain minimum of imagination. Bidder's son, it will be remembered, who was of strongly visual cast, could "conceive no other way possible of doing mental arithmetic." The elder Bidder, on the other hand, can discover in himself[4] "no particular turn of mind, beyond a predilection for figures, which many possess almost in an equal degree with myself."

(3) There is a notable absence of visual metaphors in Bidder's descriptions of his calculating operations, and a strong insistence on the *successive* character of the processes involved; whereas a visual calculator, in an account so full and clear as Bidder's, could hardly have avoided at least a few visual comparisons, and would not have laid stress on the serial, one-thing-at-a-time character of the processes. Bidder speaks[5] of

[1] *Proc. Inst. C. E.*, XV, p. 258.
[2] *Ibid.*, p. 278.
[3] *Proc. Inst. C. E.*, XV, pp. 252, 253, 256, 261, 278.
[4] *Ibid.*, p. 253. [5] *Proc. Inst. C. E.*, XV, p. 254.

"registering" the numbers in his memory, not of seeing them; in mental calculation "every mental process must be analogous to that which is indicated in working out algebraical formulæ," and, "no one step can be omitted." In reading the page of a book, "every letter of that page passes in review through the mind"; note the abstractness of the phrase here, where a visual figure would be almost inevitable for a man of visual type. Ideas of numbers are best impressed on the mind[1] "without any reference to symbols"; written figures are[2] "unmeaning symbols." As soon as we attempt to go beyond 3-figure multiplications to those of 4 figures, "another idea must be seized by the mind", namely, the idea (word) "thousands," and "this increases the strain on the registering powers of the mind."[3] In multiplication, one fact and only one is to be kept in mind at each stage.[4] Bidder would[5] "despair of any great success in the pupil's progress in the science of arithmetic if he did not commence before he knew anything of symbols, and if his first conception of numbers was not derived from their real tangible quantity and significance." Here, as frequently throughout the paper, the written symbols (visual and simultaneous) are contrasted with the "significance" of the numbers, *i. e.*, their place in the (successive and auditory) series of numbers used in counting.

In the one place (p. 255) where Bidder does use a visual comparison, it is the hackneyed one of a flash of lightning—an expression which has passed so completely into the every-day vocabulary that its original figurative force is seldom distinctly realized by those who use it. Bidder employs the phrase simply to show the rapidity and clearness with which needed numerical ideas come into his mind; this no more indicates visual type (where the counter-evidence is so strong) than the familiar phrase "like a flash" proves that things "understood" in that way are "seen" in visual terms. In fact, the verb "see" and the commoner stereotyped visual metaphors ("clear," "like a flash," etc.) are so firmly established as synonyms for "understand," etc., that, unless corroborated by more definite evidence, they prove nothing at all concerning the memory type of any one who uses them.

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[1] *Ibid.*, p. 263.
[2] *Ibid.*, p. 261.
[3] *Proc. Inst. C. E.*, XV, p. 263. Inaudi, like Bidder an auditory calculator, always keeps in mind the names (thousands, millions, billions, etc.) of the different periods of numbers, whereas a visual calculator would be much more likely to depend simply on his visual memory of position, and drop out the names of the periods as useless, like the words "put down 3 and carry 2," etc.
[4] *Ibid.*, pp. 260, 263.
[5] *Ibid.*, p. 279.

There can be little or no doubt, then, that Bidder was an auditory calculator. Scripture's sole evidence[1] of visual type in Bidder's case consists in wrongly attributing to him the younger Bidder's remark about the only conceivable way of doing mental arithmetic. The only other apparent evidence that Bidder was a visual calculator is the fact that he had good powers of visualizing diagrams and the like.[2] But we are not trying to determine Bidder's memory type in general, but only his type as a calculator; and the evidence in the latter field is practically conclusive in favor of auditory type. If we knew absolutely nothing else about Bidder's memory type in general except that he was a good visualizer of diagrams, that fact might establish a very slight presumption in favor of visual type even as a calculator; but in the presence of more definite evidence the fact is quite irrelevant here, except in so far as it shows that he was not, like Ferrol, of *extreme* non-visual type outside of calculation.

What, now, of *Colburn's* memory type? In Gall's account[3] we are told that "in calculations at all complicated, he [Colburn] is often heard to multiply, add, or subtract, aloud, and with incredible rapidity." In the *Philosophical Magazine*,[4] too, we find a reference to the "motion of his lips while calculating," and this we may take as practically conclusive evidence of auditory (auditory-motor) type. As corroborative evidence we may point to the close resemblance of the three calculators Bidder, Safford and Colburn, all of whom learned to calculate before learning to read and write, and showed a marked preference for a class of problems (evolution and factoring) not cultivated to an equal extent by any other recorded calculator. All of them began multiplication at the left, and in general had the same methods of calculation. Two of them, Bidder and Safford, we have already found to be auditory, and in Colburn's case the reference to the motion of his lips is strongly in favor of the auditory theory. Moreover, we find in him the same nervous contortions which are recorded in the case of Safford, and, to a less extent, of Inaudi,[5] both auditory (-motor) calculators.

In Colburn's case, as in Bidder's, there is one piece of apparent evidence of visual type. We read in Gall's account[6] that Colburn "was asked how he made his calculations. He an-

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[1] *Op. cit.*, p. 57.

[2] *Spectator*, LI, 1878, p. 1634.

[3] *Op. cit.*, V, p. 85.

[4] Vol. XL, 1812, p. 122.

[5] Binet says of Inaudi (*op. cit.*, p. 37), "Pendant les calculs, il fait différents gestes, tics sans importance et du reste très variables."

[6] *Op. cit.*, V. p. 87, quoting from *Med. and Phil. Jl. and Rev.*, 1811; Scripture, quoting the remark, gives the reference as p. 22 of that journal.

swered, that he saw them clearly before him." But, as we have already seen, the use of the words "see" and "clearly" is not a satisfactory proof of visual type, especially when strong and definite evidence points the other way. If Colburn had depended on Proctor's dot-patterns, that fact ought to be evident from his account of his methods in the *Memoir*, whereas we find there no reference to visual images. That Colburn saw written figures is hardly probable, since it was only in London, when almost at the height of his calculating power, that he learned to read and write. Furthermore, when the 1811 article was written on which Gall's account is based, Colburn was only 6 years old, and hence would naturally use concrete language (visual metaphors) rather than abstract language,[1] whatever his memory type. In the light of all the available evidence, therefore, it seems safe to call Colburn an auditory calculator, though it is of course just possible that at first he had visual associations also, which later may have disappeared more or less completely from his calculations.

Let us next consider the case of *Buxton*. His ability to repeat numbers either forwards or backwards establishes a slight presumption, as we have seen, in favor of visual type. On the other hand, there is no lack of presumptive auditory indications and points of resemblance to auditory calculators. Buxton not only learned to count before learning to read and write; he never learned to read and write at all. He not only, like Inaudi, retained the names of the different periods (thousands, millions, etc.) in thinking of numbers, but invented peculiar names of his own for the earlier periods of very large numbers. Like Inaudi, again, he began calculating by "naming the several figures distinctly one after another, in order to assure himself of the several dimensions [the problem was in mensuration,] and fix them in his mind."[2] Like Bidder, Colburn, Inaudi and Safford, he began at the left in multiplication; whereas the two unmistakably visual calculators, Diamandi and the younger Bidder, are known to have used cross-multiplication. He retained a marked fondness for that counting in the simple series of natural numbers with which the auditory calculator always begins; at the theatre, for instance, he counted the exits and entrances of the different characters, the words spoken by each, the number of steps

[1]Assume for the moment that a child of 6 is an absolutely non-visual calculator, and is asked to describe his methods; what language has he at command *but* the visual metaphor of "seeing clearly"? He does not yet distinguish words from thoughts, hence cannot say, as a grown man might, that he "hears" his calculations, or "speaks" them mentally. The psychology of a young child, like popular psychology in general, inevitably tends to explain everything in visual terms.

[2]*Gent. Mag.*, 1751, p. 61.

taken in the dance, and the like. His mental record of all the free beer he had had from the age of 12 must have been at least partly in verbal (auditory) terms. His skill in estimating areas by pacing them does not indicate any visualizing ability, but rather a step of uniform and known length and an ability to count and calculate. Like Inaudi and a few other calculators, none of them known to be visual, he could carry on a conversation while computing. And when a new style of problem (cube root) was proposed to him, he was heard to mutter to himself, after puzzling over it for some time, that he would master it yet.[1]

The vast extent of some of Buxton's calculations, to be sure, suggests Dase, the only other calculator known to have gone beyond 15-figure multiplications; but, even granting that Dase was a visual calculator, the value of this analogy is destroyed by the fact that whereas Buxton required two months and a half for a 39-figure multiplication, Dase performed a 40-figure multiplication in 40 minutes. And of course if Buxton had possessed anything like Dase's power of instantaneous visual counting, such trivial occupations as counting words and steps could scarcely have retained any interest for him. On the whole, then,—since the ability to repeat numbers backwards is the only indication of possible visual type, and since we can easily explain that ability, in a man whose mental processes were all so slow, by supposing that he became interested in reversing numbers and practiced the feat with his usual perseverance,—it seems fairly safe to call Buxton an auditory rather than a visual calculator.

The remaining calculators may be dismissed somewhat more briefly, since in the absence of more definite information than we have it is only possible to indicate probabilities as established by comparison with better known prodigies. *Fuller* so closely resembles Buxton that if we call the one auditory, we may assume that the other was likewise. *Mondeux* has various points of resemblance with Inaudi on the one hand, and with Bidder on the other, both of whom we have found to be auditory, hence Mondeux also may be placed in this class. *Ampère*, like Mondeux and Bidder, used pebbles in his early calculations, an indication which we have found to be no disproof of auditory type; like them, again, he learned to count at an early age,—earlier, in fact, than either of them,—before learning to read and write. A presumption therefore exists that he, too, was auditory. *Gauss* began earliest of all, before he was quite 3, if we accept his own account, as it seems safe to

[1] *Gent. Mag.*, 1753, p. 557. The exact wording of the passage is, "after some time, he said to himself *there were nooks in it* [the problem], *but he would sift them about.*"

do in the case of a man of his scientific eminence, who had so little need to exaggerate his own ability; moreover, the fact that at the gymnasium he mastered the classical languages "with incredible rapidity" suggests auditory rather than visual characteristics. Concerning the *minor prodigies* it would hardly be safe to speculate; we have seen, however, that many of them learned counting before reading and writing, and this establishes in these cases a slight presumption of auditory type.

These conclusions, it will be seen, involve a radical departure from the current views. Scripture names three calculators—Buxton, Colburn, and Bidder—as using visual images; and while he admits the absence of specific information in the case of other calculators, he thinks it safe to assume that visual images played a considerable part in all cases. We have seen, however, that the evidence in each of the three cases named is either based on a misconception, or counterbalanced by stronger evidence of auditory type. On the other hand, two of the prodigies in Scripture's list, Dase and the younger Bidder (who obviously should have been named, since he and not his father was responsible for the remark quoted by Scripture), show some evidence of visual type, and of the three others—Inaudi, Diamandi, and Zaneboni—since described, Diamandi is unmistakably visual, while Zaneboni is rather doubtful. Apart from these four men, we find in almost every case not only no evidence supporting the visual theory, but at least some evidence, of more or less weight, favoring the auditory theory. In other words, if the conclusions here reached are sound, mental calculation is primarily an auditory operation, a matter of verbal associations, and in the majority of cases no appreciable aid is rendered by visual associations. In fact, while it would perhaps be going too far to say that the "natural" calculator is always the auditory calculator, and that the visual calculator, just in so far as he *is* visual, is an "artificial" calculator, such a statement would be very much nearer the truth than the current view that mental calculation is primarily a visual operation.[1]

In case this view seems unduly radical, however, it must be remembered, in the first place, that we are dealing only with mental calculation as such, leaving quite open the question of general memory type; and in the second place, that we have

[1] Diamandi, we have seen, learned mental calculation only after leaving school, and used cross-multiplication, which is not a "natural" method. The younger Bidder used cross-multiplication and a mnemonic system, and may have had no "natural" gift for mental calculation at all, but simply have taught himself by deliberate practice, in imitation of his father, or by his father's suggestion or guidance. The cases of Dase and Zaneboni, cannot profitably be discussed from this point of view in the absence of fuller information than we possess.

adopted a very narrow definition of the two types. By calling a calculator auditory we do not imply that visual images are by any means wholly absent from his mind during calculation, but only that they are not uniformly present, and that their absence is not an embarrassment, or their presence a material aid, to the calculator. Probably in most cases what we have called chance visual associations will enter in to a greater or less extent, especially where the numbers are presented to the calculator in written or printed form. And it may conceivably happen that a man who is, in general, predominantly visual may be auditory, in this sense, as a calculator. Yet after all qualifications have been made, the fact remains that mental calculation, as we have seen again and again, naturally takes its rise from counting, which is essentially a verbal process. However widely the extent and relative weight of visual and auditory elements may vary, it can hardly conceivably happen,—certainly does not happen in any known case,—that the verbal elements play no part whatever in the calculating process; whereas the function of the visual elements may well be reduced practically to the vanishing point. And the sooner we get rid of the old idea that mental calculation is in essence the same as written calculation, but with a mental instead of a written tablet, on which the faculties of memory and imagination inscribe the figures of the written calculation in perfect order and complete detail, the sooner we shall begin to understand the psychology of mental calculation.

*Summary.* The results of our study of the mathematical prodigies may be summarized as follows:

(1) In Part I is presented a fairly complete list of the more important prodigies on record, with those data in each case which shed most light on the nature and development of the calculating power. An effort is also made to correct several errors which have crept into the literature, and to bring out a few points whose significance has been overlooked; so that the present account, it is hoped, will be found reasonably complete and reliable. In particular, the case of Zerah Colburn has been entered into in some detail; this man has certainly received less than his due from both Scripture and Binet.

(2) In Part II is described a new case, that of the present writer. The calculating power is in this case very slight, and in itself unimportant. It has been described at some length, however, to bring out the naturalness of the "precocity" involved, the gradualness of the development of the calculating power, the important part played by counting,—first in the series of natural numbers, and later in the series of multiples and powers of various numbers,—the general character of the numerical properties brought to light in this way,—properties

which awaken the interest of the calculator, furnish the motive for continued practice, and shorten the labor of calculation, rendering the whole process self-sufficient and independent of outside aid,—and, to some extent, the nature of the psychological and mathematical processes and short-cuts used in this particular case. The part played by "2-figure endings" has been entered into in some detail, as shedding light on a class of problems—evolution and factoring—which have often puzzled students of the subject, owing to the surprising rapidity with which some of the prodigies have solved them. In none of these respects is the material here presented wholly new. Most of it, perhaps, could be deduced from a careful study of Bidder's case. At the same time, it remains true that the significance of these facts has *not* been fully brought out by previous investigators; hence it has seemed worth while to dwell on them here at greater length than would otherwise be necessary.

(3) The data thus collected have been studied in Part III, and conclusions drawn under several heads. Attention has in the main been confined to the "natural" calculators who develop spontaneously, at least in the first instance, without external aid from books and teachers; as distinguished from the "artificial" calculators who use external aids from the start. Huber's blind Swiss is perhaps the only "artificial" calculator here considered; but the distinction must not be too sharply drawn, since several of the others have made use of "artificial" methods, such as mnemonic systems. A calculator may begin in the "natural" way, but later make use of "artificial" methods besides, in order to extend his calculations further, as Gauss did in his use of logarithms; if the younger Bidder had any "natural" gift to start with,—a question we must here leave open,—he belongs to the same class with Gauss in this respect. In general, it is obvious that skill in calculation attained by using these "artificial" methods, either from the start or at a later stage, constitutes no special problem for the psychology of mathematical precocity, belonging rather to the psychology of deliberately practiced operations in general, and hence need not be discussed in such a study as the present.

Into the question of *heredity* in mental calculation we have not attempted to enter, not only because of the scarcity and uncertainty of the. data, but because such general terms as heredity and environment do not carry us far in the study of any special function like mental calculation.

*Precocity* in calculation, we have found, is natural and normal; not only is the popular amazement over it groundless, but there is no need even to regard it as "remarkable." Owing to the origin of mental calculation in ordinary counting, and the complete independence and self-sufficiency of mental

arithmetic, mere mathematical precocity falls in a different class from musical precocity, and still more from the all-round precocity shown by such men as Ampère and Macaulay. If for any reason the mathematical prodigy loses his interest in calculation, or the opportunity to practice it, his power is likely to diminish or eventually to disappear; in this respect mental calculation is like piano-playing, or any other highly specialized activity dependent on long practice.

Skill in mental calculation is, owing to the isolation of mental arithmetic already noted, independent of general *education;* the mathematical prodigy may be illiterate or even densely stupid, or he may be an all-round prodigy and veritable genius. Furthermore, mental calculation is entirely independent for *mathematical* ability and education; the calculator may never rise above the counting stage, or may acquire merely arithmetical skill, or may develop a keen insight into algebraic relations, or even, like Safford and Gauss, a marked aptitude for higher mathematics. Hence it is not helpful to classify the mathematical prodigies either by general education or by subsequent mathematical development. Indirectly, however, ignorance favors a high development of the calculating power, by preventing other and more important interests from taking its place. Where the power *is* retained in spite of the widening of interests, its practical value may become considerable, especially to the mathematician, the lawyer, or the engineer.

Although mental arithmetic naturally develops out of *counting*, the fundamental operation is *multiplication*. This operation may remain permanently in the counting stage, *i. e.*, may proceed without the use of a memorized multiplication table; usually, however, a multiplication table up to 10 x 10 is used. Many of the prodigies begin at the left in both multiplication and addition. Proctor describes in his own case a method by the aid of visual dot-patterns, but no other calculator is known to have followed this method. Two of the visual calculators, Diamandi and the younger Bidder, used cross-multiplication. The theory has been proposed that a large multiplication table, perhaps to 100 x 100, is used by some of the prodigies; but there is no evidence that any of them actually did use such a table, and even Dase's feats are explicable without presupposing it. Problems in square and cube root (especially of exact squares and cubes) and factoring are favorites with some of the calculators; in those cases where the answer is given "instantly," the simple properties of 2-figure endings are used.

Various mathematical and psychological *short-cuts* explain the speed attained by some of the prodigies in their menta operations. Problems done "instantly" are either very simple, or else are solved by guess and trial, with the aid of little

tricks and properties readily discovered by the calculator. Many algebraic problems are thus solved, and the same is true of square and cube roots of perfect squares and cubes, as we have just seen.

The distinction which is so often made between *memory* and *calculation*, with the implication that the great calculator is simply a little calculator with a big memory, using the same methods as his lesser rivals, is misleading; the process is always (in the "natural" calculators) a true calculation, and memory for figures is important only in so far as it stands in the service of calculation.

Finally, many of the calculators heretofore supposed to be of visual *memory type* turn out, on closer examination, to belong to the auditory (or auditory-motor) type, at least in calculation; and, in general, since counting is essentially a verbal process, the calculator who begins from counting, before he learns to read and write, will usually belong to the auditory type, and will make relatively little use of visual images in his actual calculations. At least two of the "major" prodigies, however, and possibly four, belong to the visual type.

## APPENDIX I.

### NOTE ON ZERAH COLBURN.

Scripture, at the beginning of his treatment of Colburn (*op. cit.*, p. 11), says: "Autobiographies do not always furnish the most trustworthy evidence in regard to the man himself; when, moreover, the author is convinced that he is nothing less than a modern miracle; and, finally, when having had no scientific and little literary education, he at a later date writes the memoirs of his youth, we are obliged to supply the lacking critical treatment of the narrative." A little earlier (p. 8) he tells us that Gauss, "if he had had the misfortune to have been gifted with nothing else [than his calculating powers], . . . . might even have proclaimed himself in the Colburn fashion, as a miraculous exception from the rest of mankind." Again (p. 16), "It is to be remarked that Colburn's calculating powers, such as they were, seemed [*sic*] to have absorbed all his mental energy; he was unable to learn much of anything, and incapable of the exercise of even ordinary intelligence or of any practical application. The only quality for which he was especially distinguished was self-appreciation. He speaks, for example, of Bidder as 'the person who[1] approached the nearest to an equality with himself[2] in mental

[1] The words "in the writer's judgment" are here omitted by Scripture without the customary sign of omission. (*Memoir* p. 175.)

[2] Colburn's word here is "him," not "himself." (*Loc cit.*)

arithmetic.' Again, 'he thinks it[1] no vanity to consider himself first in the list in the order of time, and probably first in the[2] extent of intellectual power.'"

In similar strain Binet says (*op. cit.*, p. 9), "L'histoire de Zerah Colburn serait extrêmement intéressante si elle reposait sur des documents dignes de confiance; il n' en est malheureusement pas ainsi. Le principal document qui reste de lui est son autobiographie, et comme il s'est exhibé dans des représentations publiques, et qu' il parle de lui-même avec une vanité insupportable, on peut supposer à bon droit que cette biographie est une réclame." And again (pp. 10-11), "Colburn a passé pour un individu d' une intelligence médiocre, et crevant d' orgueil; sa biographie en donne mille preuves naïves, et il affirme à plusieurs reprises qu'on doit le considérer comme la plus grande intelligence de la terre."[3] Other passages of a similar sort might be quoted from both Scripture and Binet, but those above given are sufficiently typical.

These statements, it will be seen, are plain and unqualified. If they are true, Colburn was a man of little or no education, incapable of ordinary intelligence, utterly unqualified to write a historical document; and his *Memoir* is historically unreliable, an exhibition of intolerable vanity, proclaiming him as a modern miracle, the greatest intellect that ever lived, etc., in the most naïve fashion. In a word, these writers portray Colburn as a sort of Buxton proclaiming himself as a Bidder or a Gauss. Let us now examine the facts, and see how far they bear out this interpretation.

An account of Colburn's education has been given in Part I of the present paper; from that account it will be seen that Scripture's statements are, to say the least, somewhat sweeping. We are interested in the matter, however, only in so far as it concerns Colburn's qualifications for writing a reliable account of his own life; and of those qualifications the *Memoir* is itself the best test. After a careful reading, the present writer finds it internally consistent, and to all appearance painstaking and trustworthy, with no aim other than to set the facts in their

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[1] The original reads, "it is no vanity", etc. (p. 176).

[2] The word "the" here is not in the original (p. 176). Thus Scripture here gives us four misquotations in three lines,—an eloquent commentary on his method of supplying "the lacking critical treatment of the narrative"! His "critical treatment" of the Bidder family has already been discussed.

[3] Binet admits (p. 1) that his historical account is superficial and second-hand, and (p. 2, note) that he has borrowed largely from Scripture; we shall therefore not, as a rule, take account of his misstatements, though they are not few. Scripture's account, however, professes to be based on the original sources, and so may properly be held accountable for its use of those sources.

true light; and externally he has found only one or two slight discrepancies between it and other available documents, none of them at all comparable to some of the inaccuracies we have found in the article on *Arithmetical Prodigies.* We may therefore dismiss this general charge, until it is backed up by specific instances of Colburn's alleged incompetence, and turn our attention to the charge of naïve vanity and self-advertisement brought against Colburn by his critics. This is the real ground on which they have sought to discredit the *Memoir.*

There are three classes of passages in the *Memoir* on which a charge of undue self-appreciation may be based. (1) Those in which Colburn speaks of his calculating power as a gift of God, and the like. (2) His section (p. 173 ff.) on "Other Calculators," in which he attempts to estimate his own place among the calculators he had met or heard of. (3) A few scattered passages in which his language is, if not vain, at least in appearance a little unguarded. Let us consider the passages in this order.

(1) Passages of the first class may be very briefly dismissed. Colburn was brought up with eighteenth century ideas of a personal Creator, and at the time of writing the *Memoir* was a Methodist minister. For him, therefore, it is the plainest and most matter-of-fact statement possible to say that his power is a "gift of God," and unless the context shows undue pride in that gift, no charge of vanity receives the slightest support from such statements. Now actually Colburn uses these religious expressions only in this matter-of-fact way; he nowhere boasts of his gift, but, on the contrary, is frankly puzzled as to the Divine purpose in bestowing it, since it remained a mere "freak of nature," so to speak, and contributed neither to his material nor to his spiritual success. (*Cf. Memoir*, pp. 165-6.)

(2) We may now examine those parts of Colburn's section on "Other Calculators" (pp. 173-8) which bear on his own estimate of himself. The section begins with an account of his early nervousness, and concludes with some generalizing reflections, so that we may, without unduly crowding our space, quote practically in full the relevant parts of the section (pp. 173-6).

"The inquiry has frequently been made whether the writer ever became acquainted with any other persons who were endowed with a gift of mental calculation similar to himself. He thinks not, as to extent of solution." Here follows an account of what he had heard concerning Buxton, but with no attempt to compare himself directly with Buxton.

"The Countess of Mansfield called upon Zerah Colburn, while he was first exhibited in London, and alluding to the singular gift he possessed, stated that she had a daughter, Lady

Frederica Murray, who was about his age, and gave indications of superior skill in figures. He was afterwards invited to call at her ladyship's residence, and found the young lady did possess a certain degree of mental quickness uncommon in her sex and years. But her elevated rank, and the necessary attention to those pursuits which were more in accordance with her station in life, probably prevented her attending to that endowment. She was afterwards married to Colonel Stanhope, and dying young, her widowed husband, after the lapse of a few years terminated his existence by suicide.

"The person who in the writer's judgment approached the nearest to an equality with him in mental arithmetic, was a youth from Devonshire county, in England, named George Bidder. This person did not begin to excite attention until after Zerah had retired from public exhibition in London, sometime in the year 1815. Bidder was at that time ten years old.[1] Having never had any acquaintance with him, the author cannot speak correctly of the manner in which his talent was first communicated and exhibited.

"The only thing he ever heard on this point was, that his father being engaged in some difficult sum, George answered it at once; that in view of his unexpected readiness, he was put to school, and considerable pains were taken to train him for exhibition. This however may be as incorrect as some of the stories in circulation relative to the subject of this memoir. It is certain, however, that in London he [Bidder] never received that general patronage which his predecessor enjoyed.[2]

"Some time in 1818, Zerah was invited to a certain place, where he found a number of persons questioning the Devonshire boy. He [Bidder] displayed great strength and power of mind in the higher branches of arithmetic; he could answer some questions that the American would not like to undertake; but he was unable to extract the roots, and find the factors of numbers. The last time that the writer was in Edinburgh, he

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[1]Bidder was not ten years old until 1816, so that Colburn, writing from memory some fifteen years or more after the event, has made a mistake of a year in one of these figures.

[2]If this sentence exhibits vanity at all, the vanity is hardly of the naïve sort of which Colburn's critics have accused him. More probably, however, he states the simple fact. The Colburns had by this time worked the London field over pretty thoroughly, and the public must have had a genteel sufficiency of calculating prodigies. Even if we assume that Bidder's father was as assiduous as Colburn's in efforts to raise money from the nobility, which is doubtful, the generosity of the noble lords may well have faltered a trifle after Colburn had both worn off the novelty of the thing and collected all the subscriptions they were willing to give. At any rate, this passage is hardly as unequivocal a proof of vanity as we need to substantiate the sweeping charges of Scripture and Binet.

was informed that the lad was in study, under the patronage of a Scotch nobleman.

"At different periods, Zerah Colburn has heard of a number of persons, whose uncommon aptness in figures rendered them subjects of astonishment to others. He thinks it is no vanity to consider himself first in the list in the order of time, and probably first in extent of intellectual power.[1] It would be very easy to indulge in speculations in regard to the increasing number of persons thus endowed; but speculation avails little in so exact a science as mathematics, and would profit nothing on the present occasion. It is his opinion that should a similar case occur again," sufficiently promising, his education should be made a matter of public interest, etc. "It then would be seen more clearly than in any other way what was the object of the gift, and if a valuable help is therein concealed, it would be made public, and thousands might share in its advantages." The remaining paragraphs of the section on "Other Calculators" consist of general reflections which do not here concern us.[2]

Colburn thinks, then, that he has never met any one with a gift of mental calculation similar to himself, as to extent of solution; that Bidder, in his (Colburn's) judgment, came the nearest to being such a person, and was superior in several respects, but was unable to extract the roots and find the factors of numbers; and that among the calculators who have been subjects of public astonishment (*i.e.*, among professional calculators, hence not including Buxton), it is no vanity to consider himself first in the order of time, and probably first in extent of intellectual power (extent of solution). Such is Colburn's estimate of himself as a calculator.

Undoubtedly Colburn was mistaken; Bidder far excelled him, even at the time of their meeting in 1818. But we cannot leave the matter thus; in order to decide whether Colburn was led to this conclusion by vanity, we must examine the grounds he gives for it. Bidder, he tells us, displayed great strength and power of mind in the higher branches of arith-

[1] This is the only passage the present writer has found that even remotely supports Binet's assertion that Colburn "affirme à plusieurs reprises qu'on doit le considérer comme la plus grande intelligence de la terre." It need hardly be pointed out that by "intellectual power" in the present context Colburn means simply "power of calculation." Note the similar references to the "mental quickness" of the daughter of Lady Mansfield above, and to Bidder's "strength and power of mind."

[2] From these extracts the reader may judge for himself both of Colburn's style and of his historical ability or inability. In these respects, as well as for the purpose of illustrating Colburn's attitude toward his own gifts, they seem to the writer to be quite typical of the whole book.

metic, and answered some questions he (Colburn) would not like to undertake; but Bidder "was unable to extract the roots, and find the factors of numbers." Now Colburn was not much of a mathematician, but one thing he did know, from the mathematicians who had examined him: up to his own time, no one had discovered any general method of finding the factors of numbers. Colburn himself had a new and original method of performing this operation very rapidly for numbers up to 6 or 7 figures, and of finding almost instantly the roots, of exact squares and cubes. He could not be expected to understand that this method (by 2-figure endings) was really trivial; he *did* know that he could solve these problems, by an original method, and that eminent mathematicians were more amazed at this feat than at any other in his repertoire. Shall we blame him, then, for considering himself superior as a calculator to any one who simply excelled him in straight arithmetical operations, and that, too, at a time when he had given up public exhibitions and lost not a little of his former skill? Colburn could appreciate his own feats, but could not adequately appreciate Bidder's compound interest method, for example; he gives Bidder full credit, however, for "great strength and power of mind in the higher branches of arithmetic," and for defeating him in the competitive test in other directions.

Viewed in this light, the passage implies simply that Colburn was honestly mistaken in spite of a sincere effort to face the facts impartially. The charge of vanity receives no support from this part of his book, the one part above all others where vanity ought to show itself. The laudatory account of Bidder in the London paper, on the other hand, can be explained either by a better realization of the difficulty of Bidder's feats, particularly that of solving compound interest problems, by faulty memory on the part of the reporter or his informant, or even by simple patriotic partiality to the English boy. The War of 1812 was still fresh in the public mind, and love for persons and things American was not strong. Since it is not until 1819, the year after the meeting between Bidder and Colburn, that we find any record of Bidder's solving problems in square or cube root, there is no reason to dispute Colburn's statement that in 1818 Bidder had not discovered the methods he afterwards applied to it. So far, then, Colburn is completely vindicated; the charge of vanity rests on misinterpretation and on a failure to take account of all the circumstances in the case.

(3) The third class of passages need not long detain us. Two of them we have already examined; those, namely, in which Colburn says that Bidder "never received that general patronage which his predecessor enjoyed," and speaks of him-

self as "first in extent of intellectual power." After what has been said, a fair-minded critic will hardly attach serious weight to either of these passages. The only other one which seems to offer any chance for misunderstanding, even to a superficial reader, occurs on p. 63. Colburn is describing the plans for the projected book for which a committee of his admirers had been attempting, not very successfully, to collect subscriptions. The book was to be "a quarto volume, with a portrait; printed on the best paper, in a style of superior elegance. How many pages they [the committee] calculated upon is not known, but it must have required a mighty mind to extract matter sufficient to be worth eight dollars, from the history of three years of a child's life, even if that child were Zerah Colburn." Here we have a passage which may at first appear egotistic; yet the context is surely not inordinately vain. If instead of the words "Zerah Colburn" we read, "probably the greatest calculator on record," we have said the worst, and simply shown another passage in which Colburn's honestly mistaken opinion of himself comes to light. It is safe to say that but for a misinterpretation of passages of the first two classes, no one would attach any special importance to those of the third class as proofs of vanity.

Furthermore, Colburn is in several passages perfectly frank in stating his own defects. In fact, one of his critics, as extreme in one direction as Scripture and Binet are in another, speaks of the *Memoir* as "an inane production, which would be tedious in the extreme except for its absurd *naïveté* and the frankness with which the author admits his mediocrity." (*Spectator*, 1878, p. 1208.) The following passage from the *Memoir* (p. 104) will illustrate the basis on which this critic rested his estimate of the book:

"At the period of his entrance at Westminster school, he [Colburn] was a few days over twelve years old—quite old for the class in which he was placed, but for that reason better able, as well as by his eight months' attendance at the Lyceum in Paris, to get speedily removed into a higher class. During the two years and nine months that he was connected with this institution, he accomplished the labor for which the boys generally spent four or five years. He learned with facility, and the continual practice preserved what he acquired fresh in his memory. It is, however, a truth which may as well be stated here as anywhere else, that the mind of Zerah was never apparently endowed with such a talent for close thinking on intricate subjects as many possess. He was not peculiarly fortunate in arriving at a result which did not readily present itself; or for which the process leading thereto was not soon discovered. It is for this reason that he has

been unable to discover a prospect of his extensive usefulness in mathematical studies, or of justifying the high expectations which many had reasonably formed on account of his early endowment, and hence he feels more reconciled than he otherwise might in abandoning the wisdom and literature of this world for the duties of his present important calling [the ministry]. While in school he generally sustained himself among the four at the head of the class; but was not remarkable either for quickness of mind or closeness of application."

Again, in trying to account for his gift, he says (pp. 165-6), "If the notoriety of his youth was designed [by his Maker] as an introduction to him in his ministerial capacity, it would be a natural expectation that his talents as a Preacher would be equal, if not superior, to the striking displays of his early precocity. This howev[er] is far from being the case."

One other possible criticism remains to be met: it may be said that Colburn's vanity is proved by the very fact of his writing an autobiography. But the plan of writing a memoir, as we have seen, did not originate with him; and it is only when (p. 165) "at length his situation has become such that an effort was necessary to obtain some pecuniary means for supporting a wife and three little girls, over and above the contributions of the people among whom he has been laboring during the past year," that "for want of any more promising employment, this has been undertaken."

The statement seems to be warranted by the passages quoted from the *Memoir*, and the facts presented as bearing on them, that Colburn was, on the whole, free from vanity, and erred only in accepting the uncritical popular estimate of himself. Even here his error was far less than we might have expected; in fact, the wonder is that a child who was so constantly before the public in his early years, so praised and marvelled at by famous mathematicians as well as by popular audiences, did *not* develop into just the sort of vain fool that Scripture and Binet have accused him of being. But if he ever had any illusions in this matter, the hard knocks of his later life effectively removed them. In the *Memoir* he stands before us as the painstaking and conscientious historian of "a very remarkable fact in the annals of the human mind"; and while he is not alway skillful, and at times becomes, it must be confessed, too much of a preacher to be an ideal historian, his book must be taken seriously, as an important contribution to the literature on mathematical prodigies.

## APPENDIX II.

### General Table.

The following table gives a bird's-eye view of the more important mathematical prodigies, for convenience in comparing the different men. In the column headed "heredity" are found such possibly relevant facts as are known about each man's parents and relatives. In the next column is a brief account of the circumstances attending the "development" of the calculating power; *precocity* here refers only to calculation, unless "all-round precocity" is specified. Under "education" are described both the general and the mathematical training of the calculator. The next column deals with the scope and methods of his mental calculations; the next, with his figure-memory (extent, etc.), and his probable memory type; a few facts about his memory in general are added in one or two cases, but usually this has not been considered necessary. In the last column are noted other peculiarities of the calculator, whether connected with his calculations or not.

It has not seemed worth while to extend the table to other prodigies, owing to the meagreness of the available data, and the sacrifice of compactness that would be involved. It is hoped, however, that the table here given will be found helpful. Every effort has been made to render it accurate, and as complete as was consistent with the desired degree of condensation.

| NAME. | HEREDITY. | DEVELOPMENT. | EDUCATION. | MENTAL CALCULATION. | MEMORY. | REMARKS. |
|---|---|---|---|---|---|---|
| TOM FULLER (1710-1790) | Brought from Africa as a slave, at age of 14. | Probably not precocious; first records of his calculations find him at age of 70. | None. | Reduction of years etc., to seconds; sum of a geometrical progression; 9 figs. x 9 figs. Very slow calculator. | Probably of auditory type. | |
| JEDEDIAH BUXTON (1702-1772) | Father and grandfather men of some education. | Perhaps not precocious; but mental free beer record dates from age of 12. | None; very stupid from childhood; grasped even mathematical problems with difficulty. Practically no interests outside of calculation. | Handled immense numbers; once squared 39-fig. no. Methods clumsy, extremely slow; never got much beyond counting stage. | Kept mental free beer record. Probably of auditory type. | Could calculate areas pretty accurately by pacing. Could calculate while working, or carry on two different calculations a once. |
| ZERAH COLBURN (1804-1840) | Son of a farmer of little education. Supernumerary fingers and toes hereditary. | Began to calculate when 5 years old. Power developed gradually, and gradually deteriorated through lack of practice. | Fairly good, but intermittent. Showed moderate liking for mathematics, but preferred languages. | Ordinary arithmetical operations, multiplication by 5 figs., square and cube roots and factors by use of 2-figure endings. | Figure memory fairly good. Of auditory type. | Supernumerary fingers and toes. Nervous contortions in his early years when answering questions. |
| HENRI MONDEUX (1826-1862) | Son of a wood-cutter. | Tended sheep at age of 7; learned mental calculation by using pebbles. Became a professional calculator. | Received private instruction in mathematics, and showed considerable aptitude up to a certain point. | Ordinary arithmetical operations, involution, simple algebraic problems, partly by algebraic methods; ingenious processes. | Memory fair for figures, poor for other things. Probably of auditory type. | Could attend to other things while calculating. |
| JACQUES INAUDI (b. 1867) | Family not talented; prenatal influence (?) | Began calculating at age of 6, while tending sheep; at 7 could multiply 5 figs. by 5 figs. mentally. Became professional calculator. | Very limited; learned to read and write at age of 20. | Subtraction with two 21-fig. nos., addition of five 6-fig. nos., 5 figs. x 5 figs., division of two 4-fig. nos.; simple algebraic problems by trial. | Auditory type; somewhat absent-minded; highly developed figure memory, but forgets unimportant figures after a short time. | Talking during his calculations slightly delays but does not confuse him. |

| NAME. | HEREDITY. | DEVELOPMENT. | EDUCATION. | MENTAL CALCULATION. | MEMORY. | REMARKS. |
|---|---|---|---|---|---|---|
| UGO ZANEBONI (b. 1867) | Mother had good memory. | Calculations began at age of 12; well developed at 14. | Fair. | Memory feats based on railway and similar statistics, evolution with aid of 2-fig. endings, also roots of imperfect powers, probably by trial and memory. | Perhaps of visual type; good figure memory; possibly has a simple number-form. | |
| PERICLES DIAMANDI (b. 1868) | Mother had good memory; a brother and a sister share his gift for calculation. | Excelled in mathematics at school, aged 7-16; discovered calculating power on entering business, aged 16. | Good; excelled in mathematics; knows five languages. | Multiplicaton up to 5 figs. x 5 figs., etc. Uses cross-multiplication. Calculations slow. | Good memory; visual type; has a number-form, and colored audition for some names. | |
| JOHANN MARTIN ZACHARIAS DASE (1824-1861) | | Attended school at age of 2¼, took to the stage as a calculator at 15; probably precocious in calculation. | Slight; stupid in everything but calculation, even including mathematics. | Practically unlimited power of handling large nos.; 100 figs. x 100 figs. in 8¾ hours. $\sqrt{100 \text{ figs.}}$ in 52 min. Computed logarithm and factor tables, etc. | Prodigious figure memory. Perhaps of visual type. | Could count some thirty objects at a glance. |
| GEORGE PARKER BIDDER (1806-1878) | Son of mason; one brother had a remarkable memory for Bible texts, another was a good mathematician; a nephew had great mechanical talent; son was excellent mathematician and mental calculator; two granddaughters were above average ability in mental arithmetic. | Learned to count at age of 6, and soon became excellent mental calculator; retained the power through life, using it in his profession. Used pebbles, shot, etc., at first. | Good; was a man of wide interests and of considerable ability, both mathematical and general. | Ordinary arithmetical operations, multiplication up to 12 figs. x 12 figs., compound interest. Roots and factors by the aid of 2-fig. endings, etc. Methods often original and highly ingenious. Calculated rapidly. | Of auditory type in his calculations, but had good visual memory for diagrams, etc. | |

| NAME. | HEREDITY. | DEVELOPMENT. | EDUCATION. | MENTAL CALCULATION. | MEMORY. | REMARKS. |
|---|---|---|---|---|---|---|
| GEORGE P. BIDDER, JR., Q. C. (the younger Bidder) (b.1837) | Son of G. P. Bidder. | | Good; was 7th wrangler in 1858. Practiced law. | Could multiply 15 figs. x 15 figs., but slowly, and with occasional errors. Used cross-multiplication. | Visual type; has number-form. Uses a system of mnemonics in his calculations. | Possibly an "artificial" calculator, in imitation of his father. |
| TRUMAN HENRY SAFFORD (1836-1901) | Father interested in mathematics, mother of nervous temperament; both had taught school. | All-round precocity; began to calculate between ages of 3 and 5; development steady; studied higher mathematics at 8, computed and published almanacs at 9 and 10. | Good; interest in all studies, but especially in mathematics and astronomy. | Ordinary arithmetical operations; multiplication of two very easy 18-fig. nos. Roots and factors by aid of 2-fig. endings. Rapid calculator. | Memory encyclopedic in scope; of auditory type. | Nervous contortions, or at least great restlessness, during calculations in boyhood. |
| ANDRÉ MARIE AMPÈRE (1775-1836) | | All-round precocity; counted with pebbles, etc., at age of 3 or 4. | Good; all-round scholar. | Specific information lacking. | Perhaps of auditory type. Very retentive general memory. | |
| CARL FRIEDRICH GAUSS (1777-1855) | Maternal uncle mechanically and mathematically gifted. | Precocious in several directions; began mental calculation in his third year, and probably retained the power through life. | Good all-round education; became a mathematician of the highest rank. | Specific information lacking; probably, like Safford, an all-round mental calculator, by natural and book methods combined. Made use of logarithms in his mental calculations. | Very tenacious figure-memory; perhaps of auditory type. | |